Global Focus on Knowledge

“Creating mathematics” lecture 10

Understanding the Orbits of Planets

Graduate school of mathematical sciences

Takashi Tsuboi


学術俯瞰講義
教科書にはのっていない数学のお話
主題科目／テーマ講義 2単位 1、2年生対象
数学を創る—数学者達の挑戦—
コーディネータ・ナビゲータ:岡本和夫（理学部）
数学はどうやって創られたか 岡本和夫(理学部)、室田一雄(工学部)
第1回 10/8 数学はどうやって創られたか
ことばを創り、世界を創る 斎藤毅(理学部)
第2回 10/15 Mathematics "On Campus"
第3回 10/22 数の体系を創る
第4回 10/29 数と図形の共進化
脳と情報の数学を創る 甘利俊一(理化学研究所)
第5回 11/5 情報の仕組み：驚き、確率、幾何学
第6回 11/12 脳の仕組み：脳内情報の表現、記憶、学習の数理
目の錯覚の数学を創る 新井仁之(理学部)
第7回 11/19 数学で探る錯視の世界
第8回 11/26 脳の中のウェーブレット
第9回 12/3 錯視が創る新たな数学—ウェーブレットからフレームレットへ—
形を理解するための数学を創る 坪井俊(理学部)
第10回 12/10 惑星の軌道を理解する
第11回 12/17 多面体の形と曲面の上の軌道の形
第12回 1/14 形の見分け方と数学の視点
文化と数学 岡本和夫(理学部)
第13回 1/21 文化と数学
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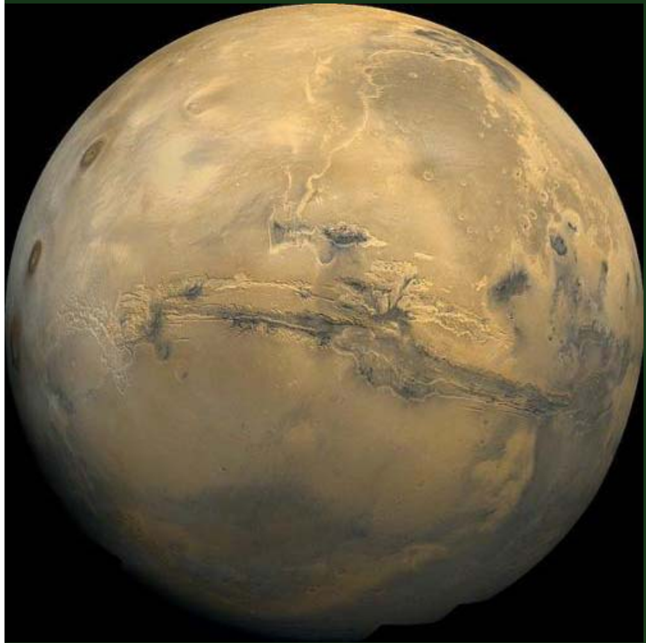

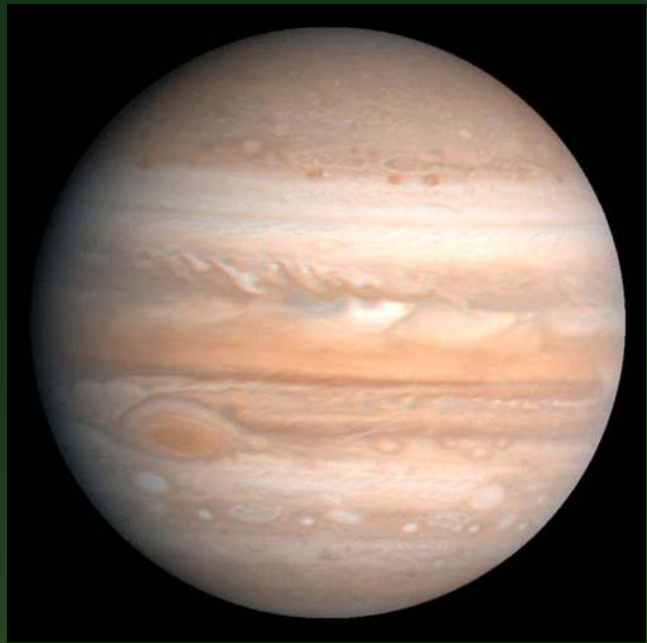

planets

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The Japanese word “惑星” is a translation of the word “planet”

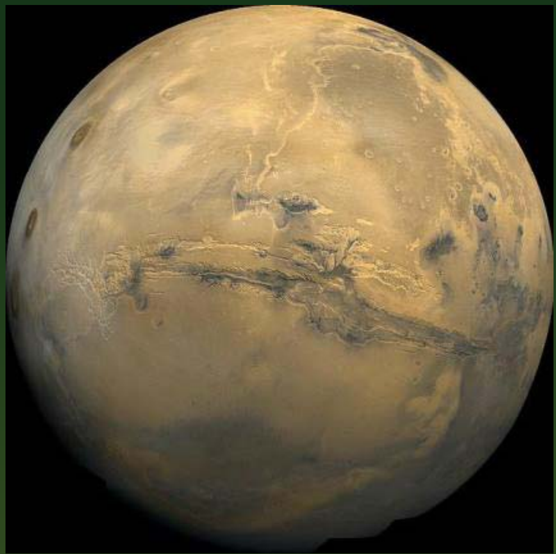
(in the 18th century, Japan)

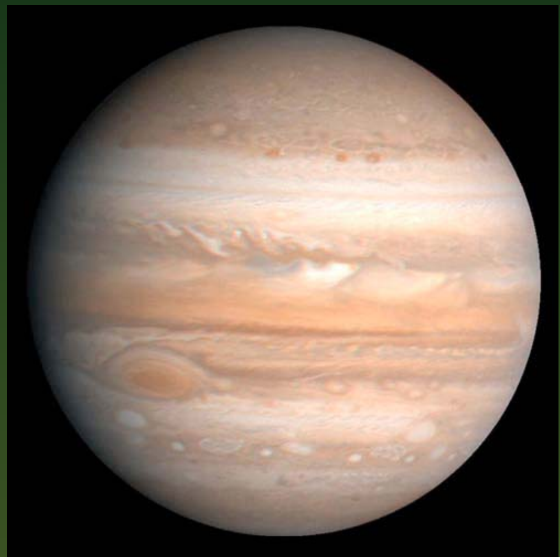
The word “planet” comes from Greek

A planet
πλανήτης a derivative of the word πλάνης meaning “moving”.

Have you tried *Mitaka*?

Today (December 10th) past 21 o'clock, Mars starts to rise in the east-northeast, and Jupiter will set in the west-southwest. Around 1 o'clock on December 11th, Saturn will rise in the east (But we can't see it due to the bright Moon nearby).





Reprinted from NASA Web page

These celestial motion must have been known for as long as several thousand years, as people looked up at the sky and made observations of celestial bodies.
I hear there is a observational record written about 3500 years ago.

It is believed that Hipparchus(around 190BC - 120BC)or Ptolemaios（around AD85-around 165) determined the table of constellations.

The name Planets come from their motion, moving from one constellation to another

Hipparchus

It was necessary for rulers to make a calendar, so that they sometimes conducted astronomical observations. To predict the motion of planets as precisely as possible was also very important for rulers.

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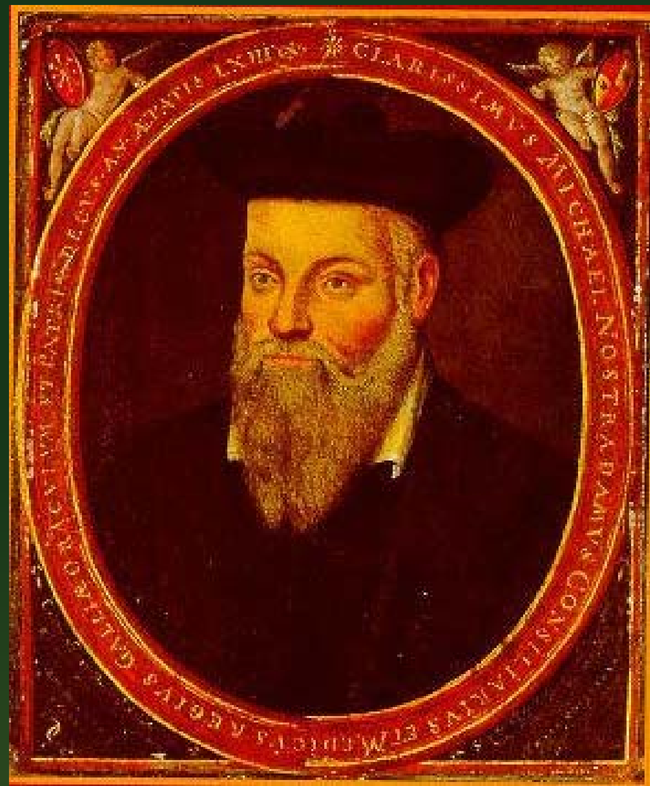
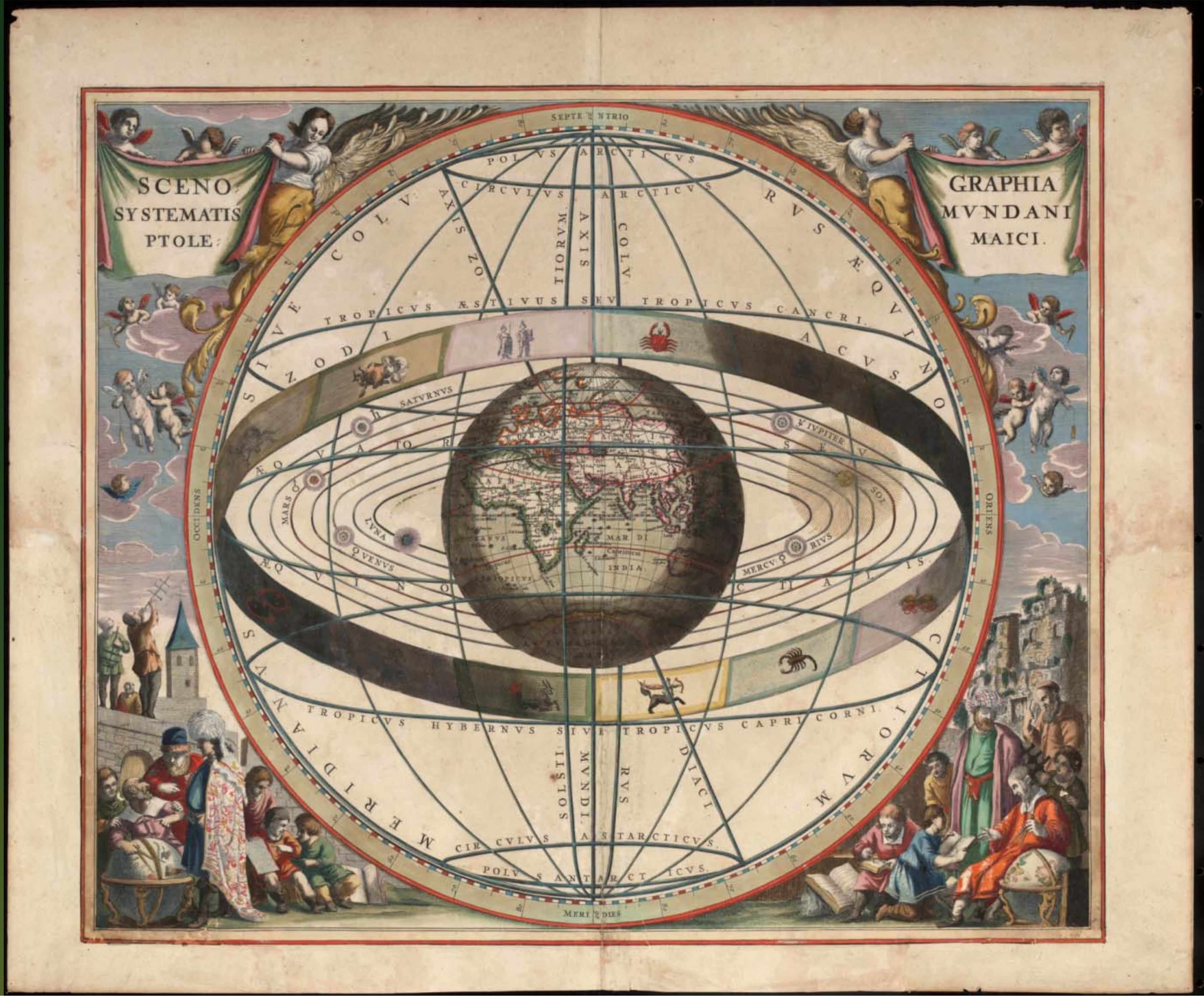


Michel de Nostredame (Nostradamus)（1503ー1566）

In astrology, the relative position of planets seems to have had some very important meaning.

Ptolemaios

In the 16th century, based on the Ptolemaios' (around AD85-around 165) geocentric theory, it was assumed that planetary motions were assumed to be on the epicycle. But even if we take the modifications into account, the observed planetary positions were found to be inconsistent with the calculated ones.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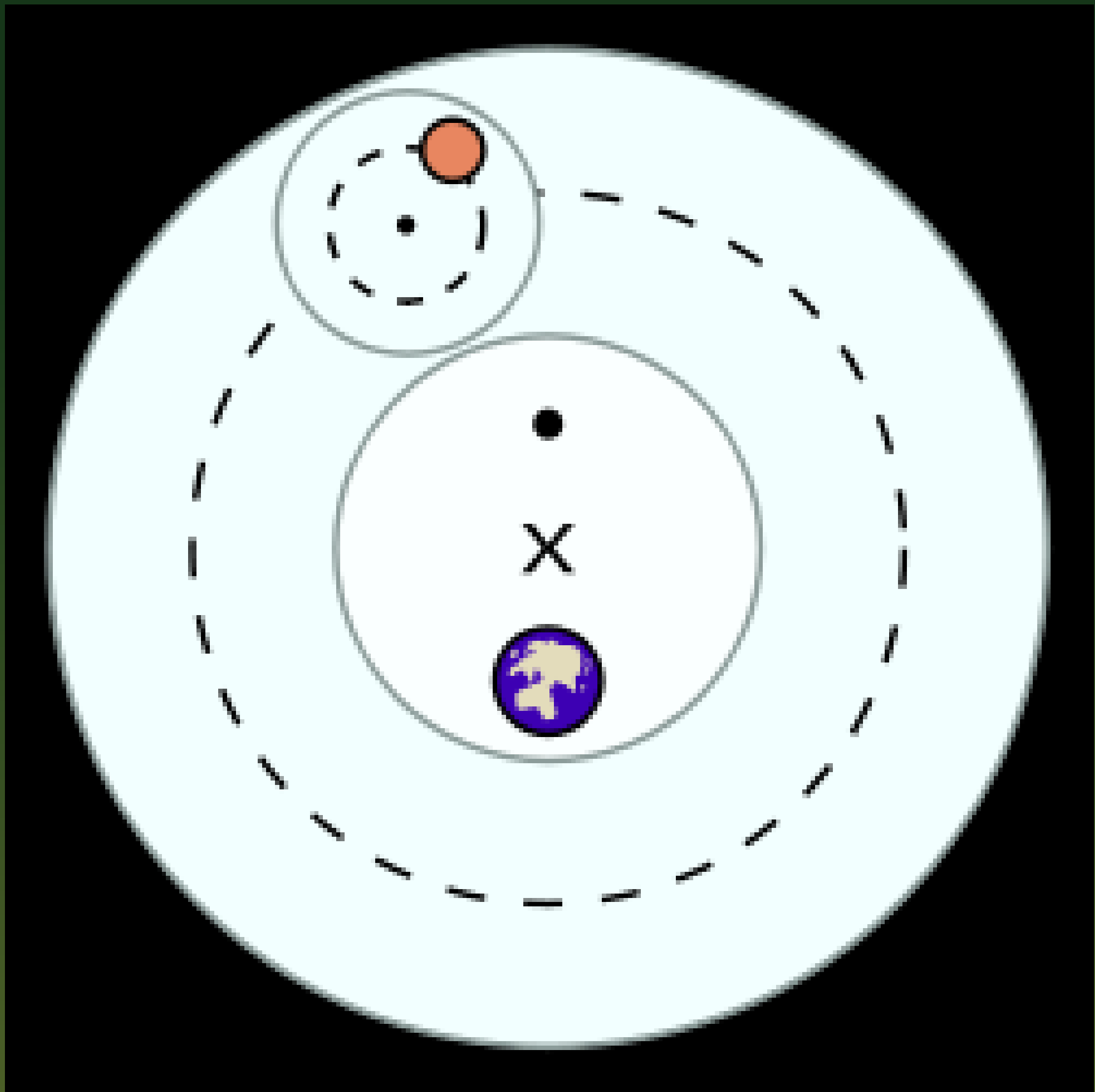
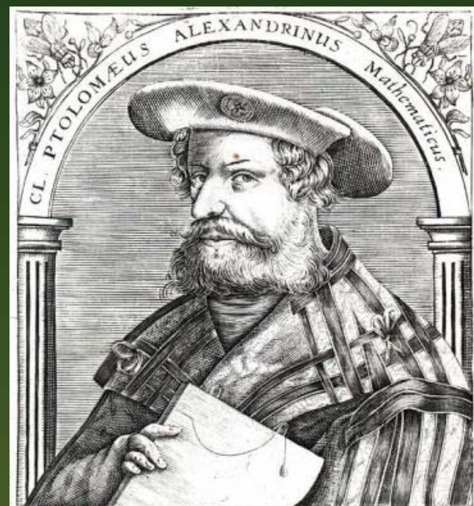
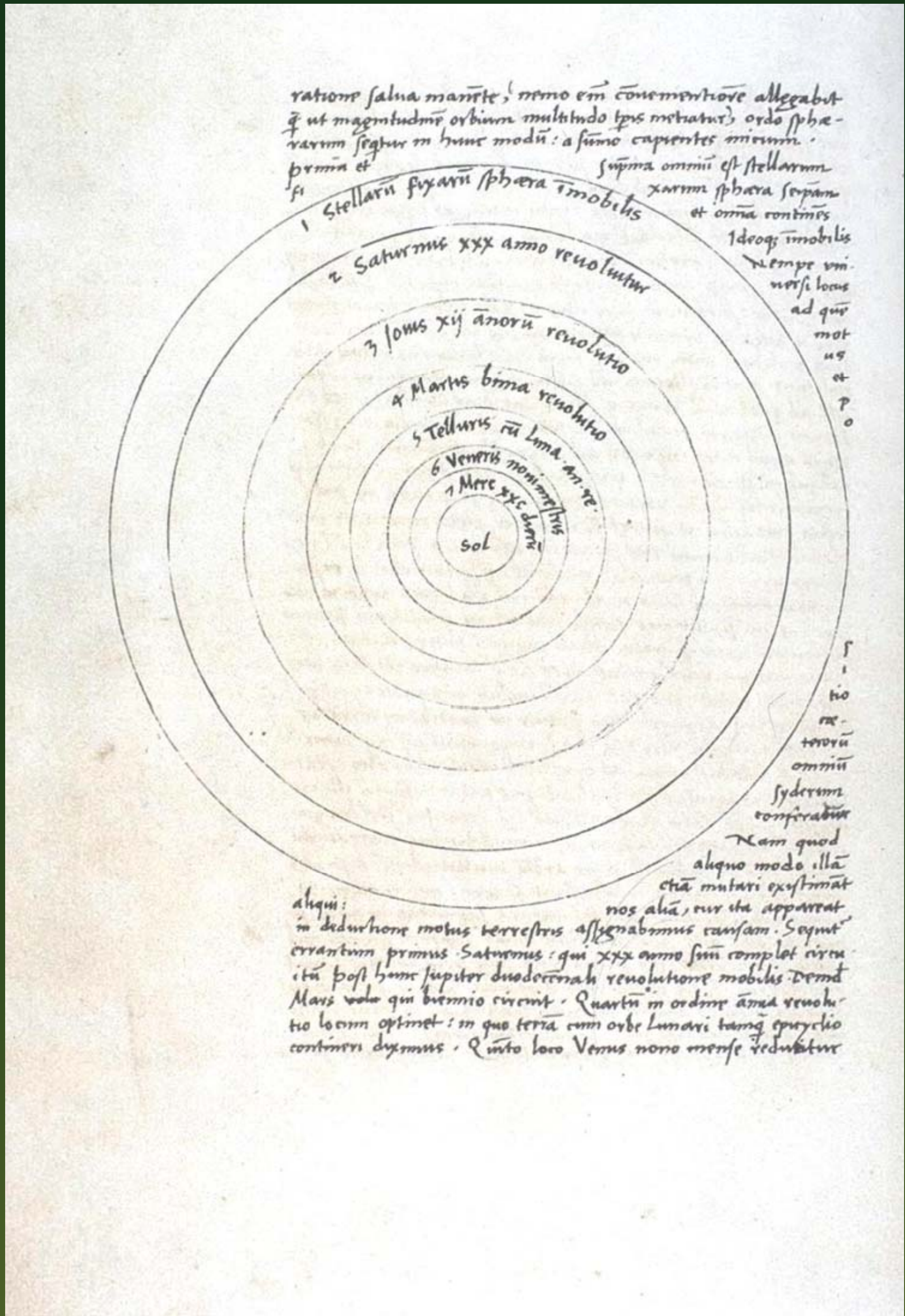
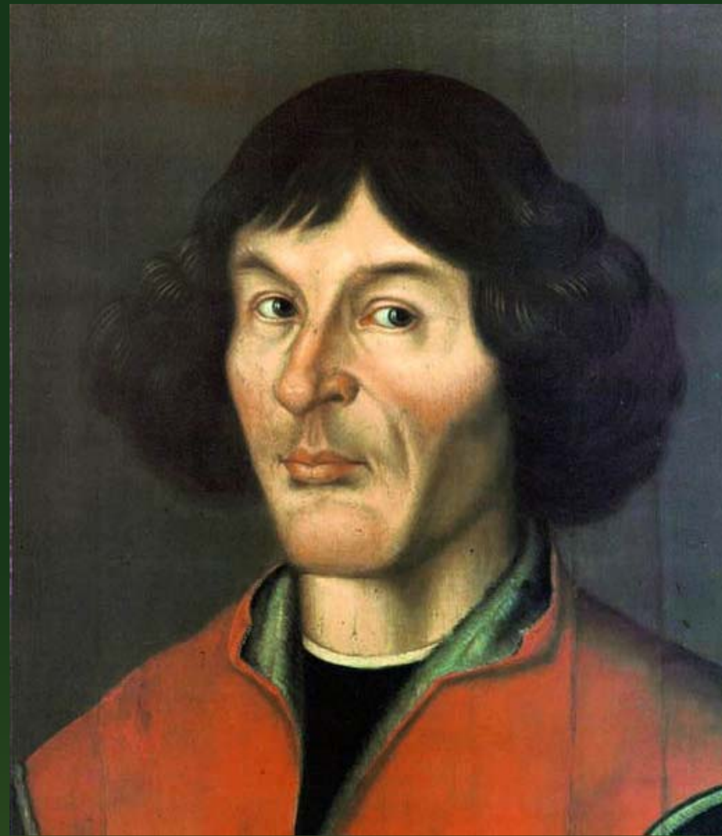


Apollonius
（262BC－190BC）



Ptolemaios
（AD85頃 －165頃）





Copernicus
（1473－1543）

- The work of precise calculation was a very strenuous work.

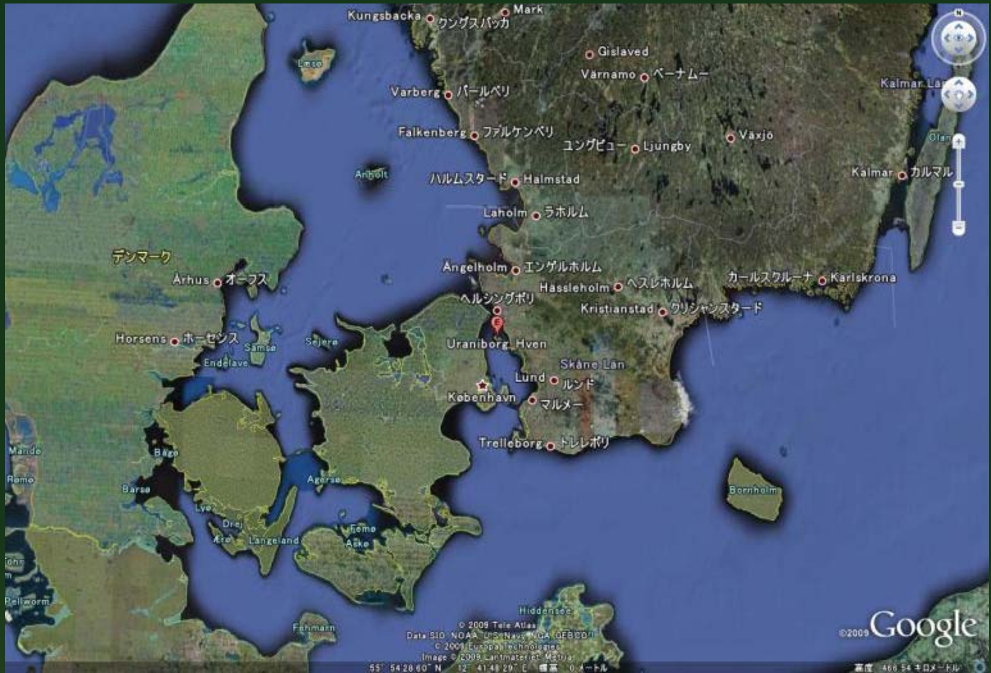
- In the same era, some of practically very important functions came into use. For example, trigonometric functions (sin x, cos x, tan x), logarithmic function (log x) discovered by Napier (1550-1617), which enabled mathematicians to grasp the meaning of exponential function e^x ,and so forth.

- Logarithmic function table and trigonometric function table had been used until quite recently.

Napier (1550 —1617)

- with the aid of King of Denmark, Tycho Brahe (1546－1601) founded an astronomical observatory at the Hven Island, where he could do his observations from 1576 to 1597.
- The astronomical observatory was a large institute, even with a printing office.
- He tried to forecast the position of planets with very precise observations.
- He determined positions of stars within the error of about 1’.
(1’=2π/21600)

Tycho Brahe (1546－1601)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Varberg バールベリ
Falkenberg ファルケンベリ
ユングビュー Ljungby
Växjö
ハルムスタード Halmstad
Kalmar カルマル
Laholm ラホルム
デンマーク
Århus オーフス
Ängelholm エンゲルホルム
Hässleholm ヘスレホルム
ヘルシングボリ
カールスクルーナ Karlskrona
Kristianstad クリシャンスタード
Horsens ホーセンス
Uraniborg, Hven
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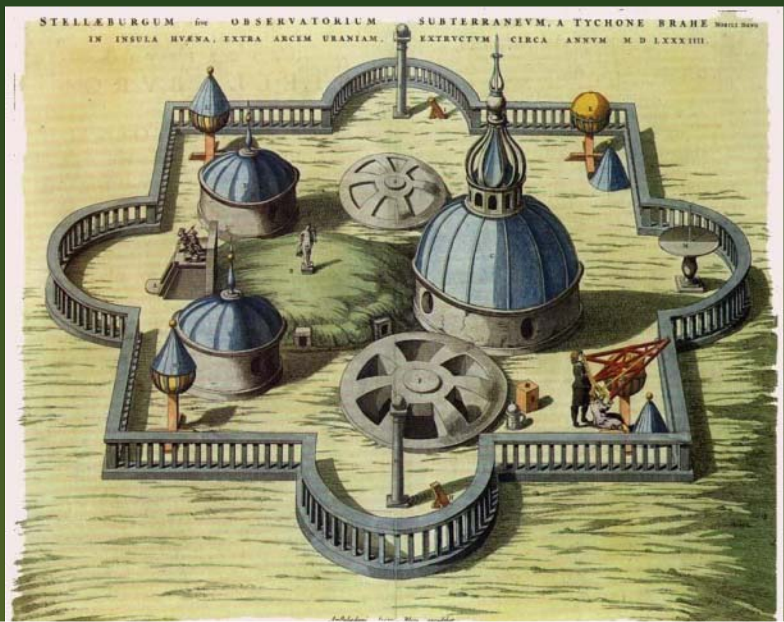
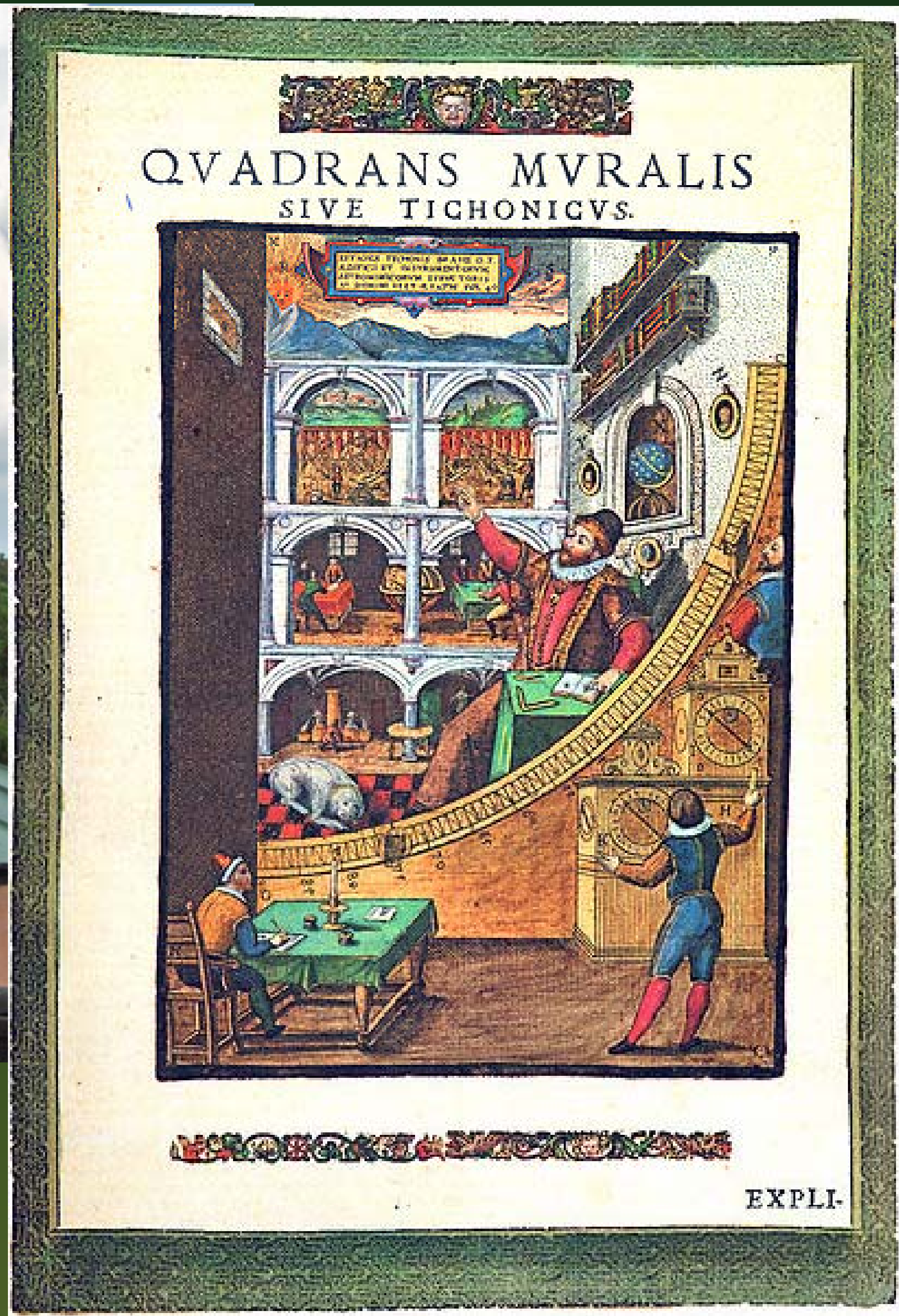

STELLÆBURGUM sive OBSERVATORIUM SUBTERRANEVM, A TYCHONE BRAHE
IN INSULA HVÆNA, EXTRA ARCEM URANIAM, EXTRVCTVM CIRCA ANNVM M D LXXXIIII.


ARCIS VRANIBVRGI, A TYCHONE BRAHE, DÑO DE KNVDSTRVP,
IN INSVLA HELLESPONTI DANICI HVENNA CONSTRVCTÆ, QUO AD TOTAM CAPACITATEM, DESIGNATIO.

Reprinted from Wikipedia
http://commons.wikimedia.org/wiki/File:StjärneborgObservatory.jpg
(2010/09/06)



Kepler (1571—1630)

- Tycho Brahe went to Prague after the death of the King of Denmark who had helped him, where he was employed by the Holy Roman Emperor and Johannes Kepler (1571-1630) became an assistant.

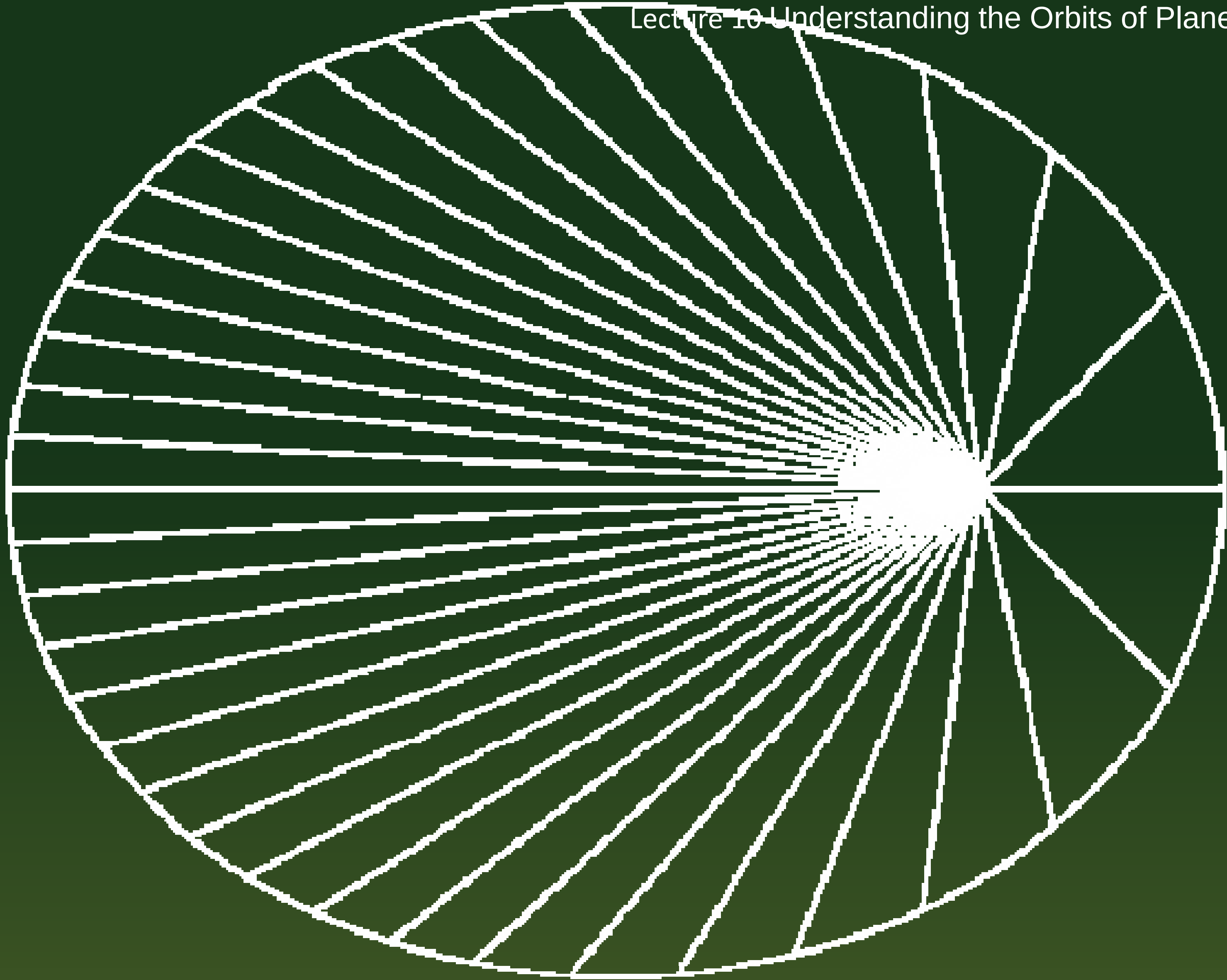
- Kepler , a mathematician, reorganized the observational data left by Tycho Brahe, and presented the results in the form of three laws, namely the first and second law (1609) and the third law (1619).

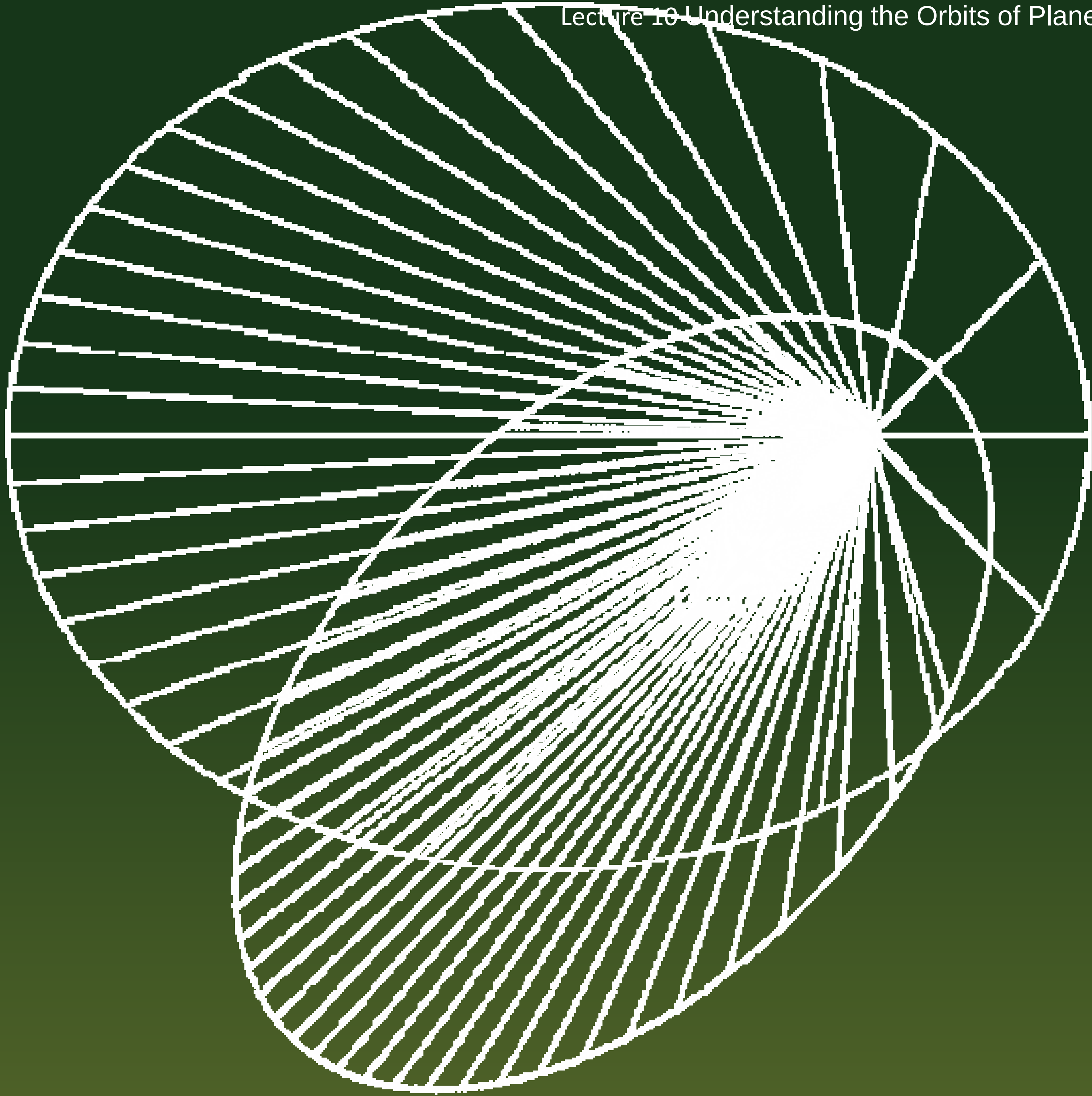
- The first law: the orbit of a planet is an ellipse whose focus is the position of the sun.

- The second law: the areal velocity of the motion of the planet is constant.

- The third law: the cube of the semi-major axis of orbital ellipse is proportional to the square of orbital period.

Kepler is the first mathematician who used the word a “focus” of an ellipse.



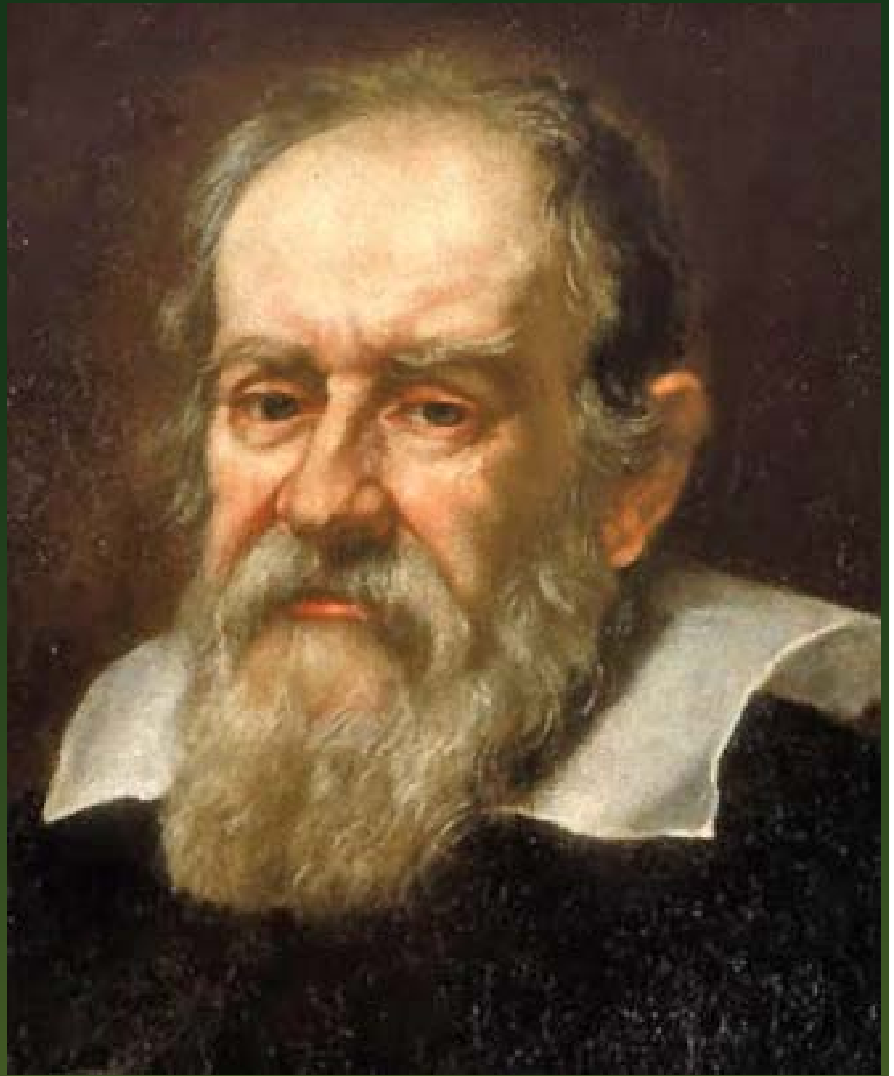


- it was widely believed that when things move, there should be always a force that drives them. Against this common sense Galileo(1564-1642) demonstrated, by an experiment, the law of inertia.

- Galileo also confirmed that for free falling particle, the distance is proportional to the square of the elapsed time from the beginning of the motion.

- In 1609, Galileo started observations of stars using telescopes.

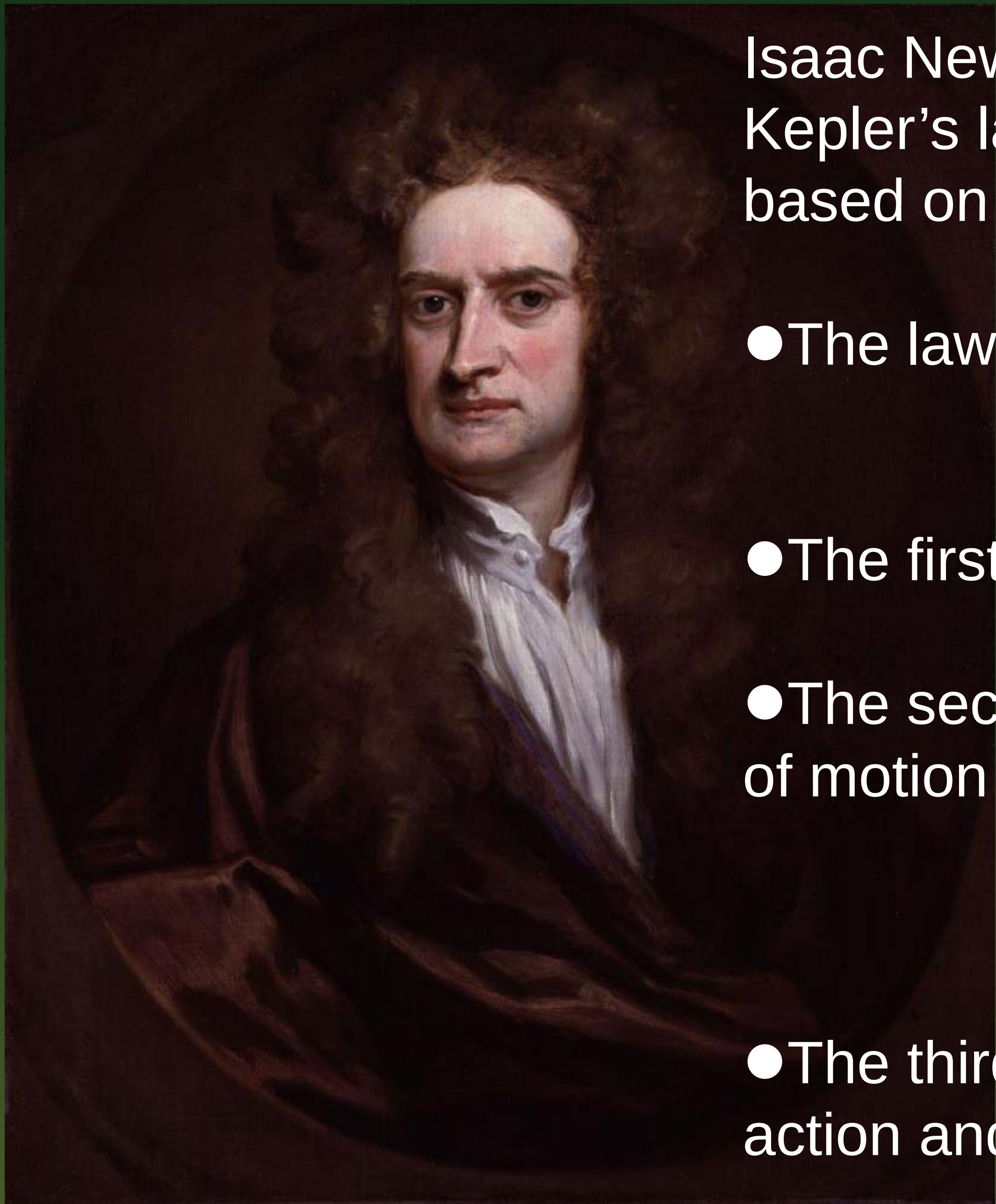
- International year of astronomy 2009



Galileo (1564－1642)

He introduced the concept of coordinates, and showed that the term of two worlds can be mutually translated, namely the algebraic world written by equations and formulas, and the geometric world written by figures.

Decartes(1596－1650)



Isaac Newton(1643－1727) explained the Kepler's laws in his book "Principia", based on the following assumptions

● The law of universal gravitation

$$\|\vec{F}\| = G\frac{mM}{r^2}$$

● The first law of motion = the law of inertia

● The second law of motion = the equation of motion

$$\vec{F} = m\frac{\mathrm{d}^2\vec{q}}{\mathrm{d}t^2}.$$

● The third law of motion = the law of action and reaction

Reprinted from Wikipedia
http://en.wikipedia.org/wiki/File:Sir_Isaac_Newton_by_Sir_Godfrey_Kneller,_Bt.jpg(2010/09/05)

Newton invented the concept of differentiation and acceleration, and derived the equation of motion.

Mathematics was created.

$$\vec{F} = m\frac{\mathrm{d}^2\vec{q}}{\mathrm{d}t^2}.$$

Newton (1643－1727)

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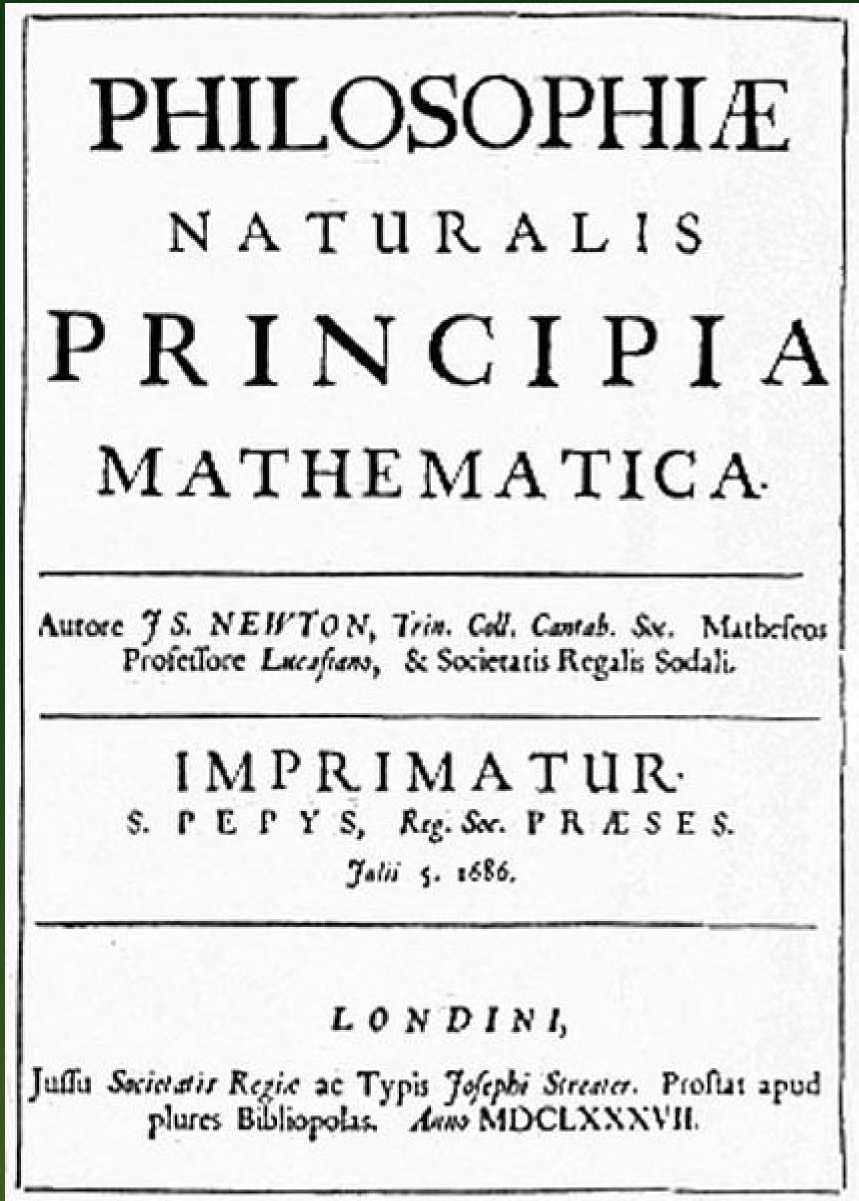
http://en.wikipedia.org/wiki/File:Sir_Isaac_Newton_by_Sir_Godfrey_Kneller,_Bt.jpg(2010/09/05)



PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

In fact, Newton wrote his compositions using the concept of differentiation and the concept of acceleration, and solved the problems with geometrical methods.

Although there are some formulas, but almost all of them are concerned with the length of geometrical figures.

Makoto Matsumoto：1999
The mathematical society of Japan annual meeting, a lecture for citizens
“it must be impossible to discover the universal law of gravity when an apple falls from a tree”

LEGES MOTUS.

LEX I.

(s) *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quâtenus a viribus impressis cogitur statum illum mutare.*

PROJECTILIA perseverant in motibus suis, nisi quâtenus a resistentiâ aëris retardantur, et vi gravitatis impelluntur deorsum. Trochus, cujus partes cohærendo perpetuò retrahunt sese a motibus rectilineis, non cessat rotari, nisi quâtenus ab aëre retardatur. Majora autem Planetarum et Cometarum corpora motus suos et progressivos et circulares in spatiis minus resistentibus factos conservant diutius.

LEX II.

(t) *Mutationem motus proportionalem esse vi motrici impressæ, et fieri secundùm lineam rectam quâ vis illa imprimitur.*

Si vis aliqua motum quemvis generet; dupla duplum, tripla triplum generabit, sive simul et semel, sive gradatim et successivè impressa fuerit. Et hic motus (quoniam in eandem semper plagam cum vi gener-

(s) 24. Ex hâc primâ lege quam (9) demonstravimus, sequitur omnem motum esse naturâ suâ æquabilem et rectilineum, adeóque nec illius velocitatem retardari, nec directionem mutari, nisi aliquod obstaculum mobili offeratur; Unde cùm projectilia motum suum sensim amittant, quærenda est aliqua hujusce retardationis causa. Cum autem corpora projecta vel per medium resistens deferantur, vel etiam super aliorum corporum superficies scabras incedant, et vi gravitatis deorsùm semper urgeantur, necesse est ut eam amittant motùs sui partem quam in hisce obstaculis superandis continuò absumunt, ac proindè quo major vel minor erit medii resistentia, eò majus vel minus decrementum accipiet corporis projecti velocitas. Ex his igitur patet majora planetarum et cometarum corpora nullam sensibilem in spatiis cœlestibus experiri resistentiam, cùm motus suos diutissime conservent.

(t) 25. Si corpus vi activâ, qualis est vis gravitatis, secundùm eandem aut parallelam directionem continuò urgeatur, motus illius continuò acceleratur; nam per leg. 1., manet celeritas acquisita, et per leg. 2. nova conspiranti continuò additur. Si verò aliqua vis in corpus jam motum contrariâ directione perpetuò agat, motus illius continuò retardatur, per leg. 2. Si vis conspirans continuò ac uniformiter agat, id est, si constans sit, corpus eâ vi impulsum, æqualibus temporibus æqualia accipit celeritatis incrementa, seu motu uniformiter accelerato fertur, et celeritates vi illâ acquisitæ, sunt ut tempora quibus

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atrice determinatur) si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel obliquo obliquè adjicitur, et cum eo secundùm utriusque determinationem componitur.

(*) LEX III.

Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.

Quicquid premit vel trahit alterum, tantundem ab eo premitur vel trahitur. Si quis lapidem digito premit, premitur et hujus digitus a lapide. Si equus lapidem funi alligatum trahit, retrahetur etiam et equus (ut ita dicam) æqualiter in lapidem: nam funis utrinque distentus eodem relaxandi se conatu urgebit equum versus lapidem, ac lapidem versus equum; tantumque impediet progressum unius quantum promovet progressum alterius. Si corpus aliquod in corpus aliud impingens, motum ejus vi suâ quomodocunque mutaverit, idem quoque vicissim in motu proprio eandem

generantur. At si vis constans contrariâ directione in corpus motum continuò agat; æqualibus temporibus æqualia fient celeritatis decrementa, et corpus motu uniformiter retardato movebitur. Generaliter tandem, si corpus quiescens quâlibet vi sive constanti sive variabili continuò urgeatur, et deinde eâ celeritate quam vis illius actione continuâ acquisivit, contrà directionem vis illius reagentis projiciatur, ut vestigia sua relegat, corpus illud in itu et reditu suo eandem habebit celeritatem, ubi ad eadem viæ suæ puncta, eundo et redeundo pervenerit; adeoque motum redeundo non amittet, nisi cùm pervenerit ad punctum ex quo cœpit eundo moveri; nam eadem vis in itu et reditu corporis, æqualibus temporibus æquales celeritatis gradus generat et extinguit (8).

26. Corpora gravia in terræ viciniis, sublatâ medii resistentiâ, motu uniformiter accelerato descendunt, et motu uniformiter retardato ascendunt . . . . . . Demonstratio . . . . Sublatâ medii resistentiâ idem est ejusdem corporis pondus, sive eadem illius in subjectum planum pressio, tum in vertice, tum in radice montis; est autem pondus, seu vis motrix (15) ut massa in vim gravitatis acceleratricem ducta: ergò cum ejusdem corporis massa eadem in vertice et in radice montis permaneat, manebit etiam eadem vis acceleratrix gravitatis. Insuper corpora gravia in radice et vertice montis æqualia spatia æqualibus temporibus percurrunt, sublatâ aëris resistentiâ, ut accuratissimis notum est experimentis (13): constans est igitur vis acceleratrix, et per lineas ad horisontem perpendiculares (3) uniformiter agit; gravia ergò motu uniformiter accelerato descendunt, et uniformiter retardato ascendunt (25). Q. e. d.

27. Sublatâ medii resistentiâ in terræ viciniis, spatia quæ corpus è quiete cadendo percurrit, sunt ut quadrata temporum quibus percurruntur . . . . Dem. . . . Recta S K, repræsentet spatium quod grave cadendo percurrit; T C, T c, T B, exponant tempora quibus describuntur spatia S P, S p, S K; et C L, c l, B D, ad T B, normales, exhibeant celeritates temporibus T C, T c, T B, per spatia S P, S p, S K, acquisitas; quia in motu uniformiter accelerato, celeritates sunt ut tempora, (25), erit T C : T c = C L : c l; et T C : T B = C L : B D, adeóque recta, T D, transit per puncta L, et l, et triangula T C L, T c l, T B D, similia sunt. Jam fingamus lineam, c l, motu sibi semper parallelo ità accedere ad lineam C L, ut tandem cum ipsâ coincidat; evanescente tempusculo C c, celeritas, c l, non differet a celeritate C L, adeóque per tempusculum infinitè parvum seu evanescens C c, celeritas, C L, uniformis censeri potest. Porrò spatia motu æquabili descripta sunt ut celeritas in tempus ducta (5), ergò spatium P p, quod tempusculo, C c, percurri supponimus, est ut rectangulum, C L × C c = C d; quare si totum tempus, T C, in tempuscula innumera ut C c, divisum concipiatur, et similiter spatium S P, tempore T C, percursum in totidem spatiola evanescentia, singulis tempusculis correspondentibus percursa dividatur, erit summa rectangulorum C d, hoc est area trianguli T C L, ut summa spatiolorum P p, id est ut S p; et eodem modo demonstratur aream trianguli T B D, esse ut spatium S K, tempore T B, percursum. Est igitur triangulum T C L : T B D = S P : S K. Sed triangulorum similium areæ T C L, T B D, sunt ut quadrata laterum homologorum.

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Quicquid premit vel trahit alterum, tantunden
hitur. Si quis lapidem digito premit, premitur
Si equus lapidem funi alligatum trahit, retrahetu
dicam) æqualiter in lapidem: nam funis utrinqu
andi se conatu urgebit equum versus lapidem, ac
tantumque impediet progressum unius quantum p
terius. Si corpus aliquod in corpus aliud impin
quomodocunque mutaverit, idem quoque vicissim

generantur. At si vis constans contrariâ directione in corpus motum continuò agat; æqualibus temporibus æqualia fient celeritatis decrementa, et corpus motu uniformiter retardato movebitur. Generaliter tandem, si corpus quiescens quâlibet vi sive constanti sive variabili continuò urgeatur, et deinde eâ celeritate quam vis illius actione continuâ acquisivit, contrà directionem vis illius reagentis projiciatur, ut vestigia sua relegat, corpus illud in itu et reditu suo eandem habebit celeritatem, ubi ad eadem viæ suæ puncta, eundo et redeundo pervenerit; adeoque motum redeundo non amittet, nisi cùm pervenerit ad punctum ex quo cœpit eundo moveri; nam eadem vis in itu et reditu corporis, æqualibus temporibus æquales celeritatis gradus generat et extinguit (8).

26. Corpora gravia in terræ viciniis, sublatâ medii resistentiâ, motu uniformiter accelerato descendunt, et motu uniformiter retardato ascendunt . . . . . . Demonstratio . . . . Sublatâ medii resistentiâ idem est ejusdem corporis pondus, sive eadem illius in subjectum planum pressio, tum in vertice, tum in radice montis; est autem pondus, seu vis motrix (15) ut massa in vim gravitatis acceleratricem ducta: ergò cum ejusdem corporis massa eadem in vertice et in radice montis permaneat, manebit etiam eadem vis acceleratrix gravitatis. Insuper corpora gravia in radice et vertice montis æqualia spatia æqualibus temporibus percurrunt, sublatâ aëris resistentiâ, ut accuratissimis notum est experimentis (13): constans est igitur vis acceleratrix, et per lineas ad horizontem perpendiculares (3) uniformiter agit; gravia ergò motu uniformiter accelerato descendunt, et uniformiter retardato ascendunt (25). Q. e. d.

27. Sublatâ
spatia quæ cor
sunt ut quadrat
. . . . Dem. . . .
quod grave cad
exponant temp
S P, S p, S K
normales, exhib
T c, T B, per
quis in motu
sunt ut tempora,
et T C : T B
T D, transit p
T C L, T c l,
mus lineam, c
accedere ad lin
coincidat; evan
c l, non differe
tempusculum in
celeritas, C L,
rò spatia motu
ritas in tempus
quod tempuscul
ut rectangulum
totum tempus,
C c, divisum c
S P, tempore T
evanescentia, si
tibus percursa
lorum C d, bo
summa spatiolo
modo demonst
esse ut spatium
Est igitur trian
S K. Sed trian
T B D, sunt ut

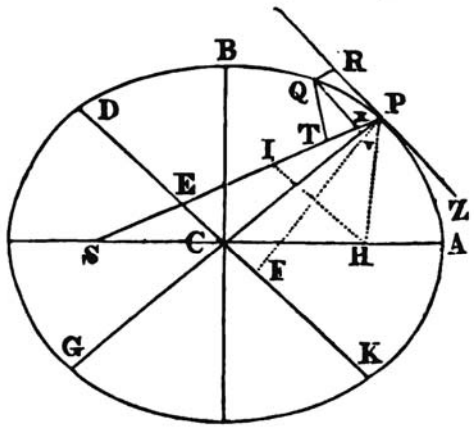
SECTIO III.

De motu corporum in conicis sectionibus excentricis.

PROPOSITIO XI. PROBLEMA VI.

Revolvatur corpus in ellipsi: requiritur lex vis centripetæ tendentis ad umbilicum ellipseos.

Esto ellipseos umbilicus S. Agatur S P secans ellipseos tum diametrum D K in E, tum ordinatim applicatam Q v in x, et compleatur parallelogrammum Q x P R. Patet E P æqualem esse semiaxi majori A C, eo quod, actâ ab altero ellipseos umbilico H lineâ H I ipsi E C parallelâ, ob æquales C S, C H æquentur E S, E I, (g) adeo ut E P semi-summa sit ipsarum P S, P I, id est (ob parallelas H I, P R, et angulos æquales I P R, H P Z) ipsarum P S, P H, quæ conjunctim axem totum 2 A C adæquant. Ad S P demittatur perpendicularis Q T, et ellipseos latere recto principali (seu (h) $\frac{2\,BC\text{ quad.}}{AC}$) dicto L, erit L × Q R ad L × P v ut Q R ad P v (i) id est, ut P E seu A C ad P C



(g) 260. Quia (per Prop. 48. Lib. 3. Conic. Apoll. sup. Theor. IV. de Ellipsi) æquales sunt anguli quos rectæ P H, P S, constituunt cum tangente P R, et ob parallelas H I, P R, æquales quoque sunt anguli alterni P I H, P H I, æquales erunt rectæ P I, P H, adeóque $E P = \frac{P S + P H}{2} = A C$, (Prop. 52. Lib. 3. Conic. Apoll. superius Theor. III. de Ellip.)

(h) 261. In Ellipsi et Hyperbolâ latus rectum principale $L = \frac{2\,BC^2}{AC}$ nam $2\,AC : 2\,BC = 2\,BC : L$, undè $L = \frac{4\,BC^2}{2\,AC} = \frac{2\,BC^2}{AC}$.

(i) Per constructionem Q R = P x, sed propter Triangula similia P x v, P E C, P x : P v = P E (A C) : P C, ergò Q R : P v = A C : P C.

Hooke(1635-1703)

- By conducting various experiments, Hooke (1635-1703) seems to have conjectured the fact that the sun is pulling the planets, and that the magnitude of the pulling force is inversely proportional to the square of the distance between the sun and a planet.
- Hooke was engaged in some experimental research work at the Royal Society, and the Hooke’s law, which says the amount of a spring’s force is proportional to the variations of length of the spring, is named after him, but it seems that the discovery of many physical laws are made by him.
- But Newton argues the presence of the universal gravitation, not only for the sun and the planets.

It was Leibniz who wrote the differentiation as $\frac{df}{dt}$, and the integration as $\int_a^b f(x)dx$

The word "integral" seems to have been invented by Jacob Bernoulli.

Leibniz used the term "summation", and the integration symbol $\int$ is deformed form of one of the alphabet S

Leibniz(1646 — 1716)

The Kepler's second law "the law of constant areal velocity" is the manifestation of the fact that as the central force is exerted on the star in motion, the star is changing its velocity in the direction of the central force.

- the reason why the areal velocity is constant has proved to be well explained if the gravitational force is assumed to be in the direction of the line joining the planet and the sun.

- The “line joining the planet and the sun” is named “vector” meaning that it conducts forces, which is used, in the same usage, in Biology.

- this object that “conducts forces” became an object called as “vector”, which is used in mathematics to indicate a quantity that has both direction and magnitude. Hamilton(1805－1865) gave the definition.

- The concept of new “vector” was established, and the mathematics was created.

The fact that the orbit of planets are ellipses is a manifestation that the universal gravitation is inversely proportional to the square of the distance between any two bodies.

A proof by Feynman:

Areal velocity is constant. i.e.
For the angular variation with the center being the sun,
r is the function of θ

$$r^2 \frac{d\theta}{dt} = \text{const.}$$

The force is inversely proportional to the square of the distance

$$F = \frac{\text{const}}{r^2}$$

Hence,

$$F \frac{dt}{d\theta} = \text{const.}$$

- From $F\frac{dt}{d\theta} = \text{const}$ and from the equation of motion $m\frac{dv}{dt} = F$,we get $\frac{dv}{d\theta} = \text{const}$
- $\left|\left|\frac{d\vec{v}}{d\theta}\right|\right| = \text{const}$,this means that the rate of the change of particle velocity doesn't depend on the angular position.
- Equivalently,

$$\frac{d\vec{v}}{d\theta} = a\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$$

If $\left\|\frac{d\vec{v}}{d\theta}\right\| = \text{const}$ and, since the motion is due to a central force,

If follows that $\frac{d\vec{v}}{d\theta} = a\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$ Then $\vec{v} = \vec{v}_{\mathbf{average}} + a\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}$,

And the orbit of the velocity $\vec{v}(\theta)$ will be a circle.

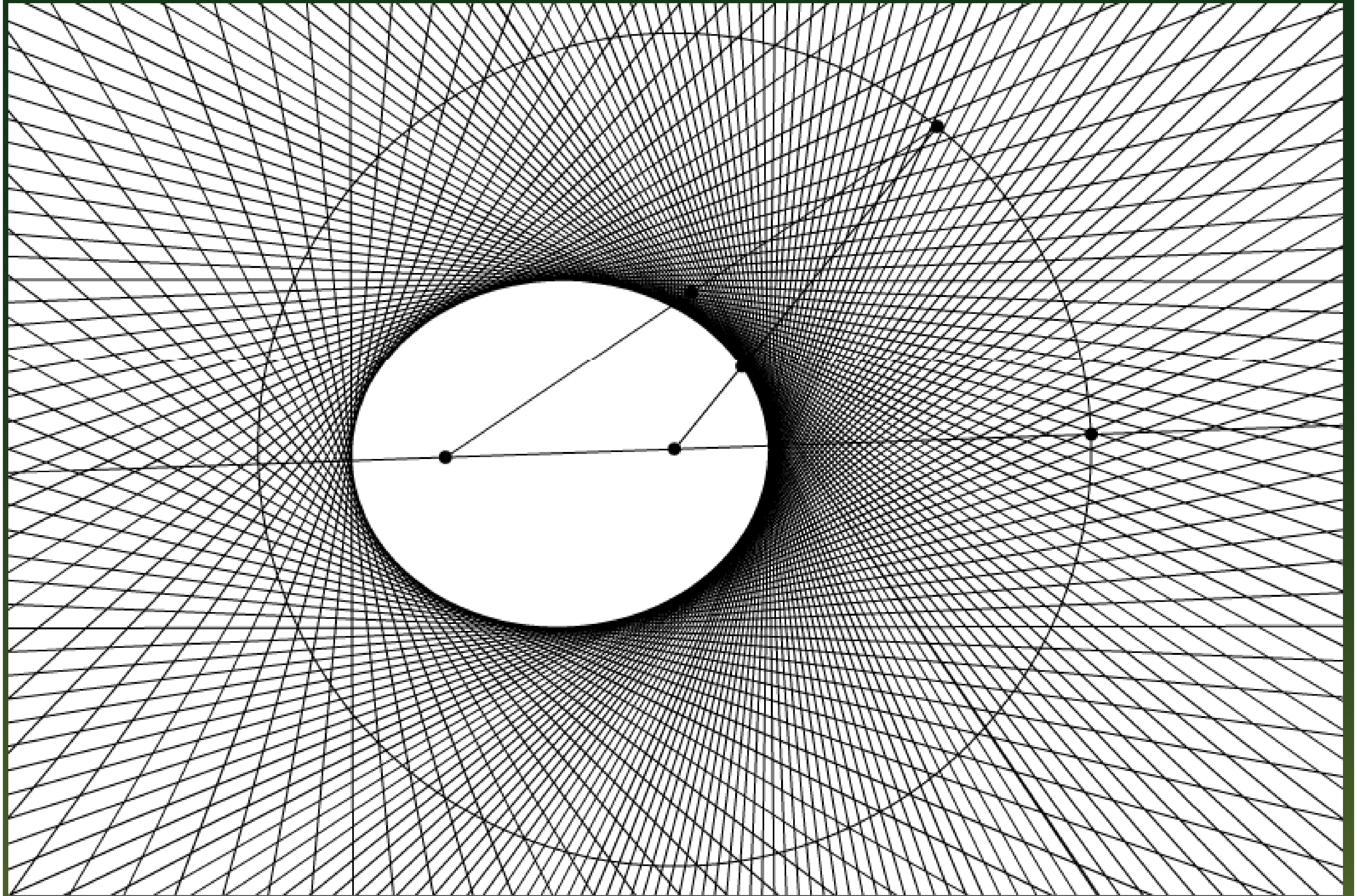
If you know nothing about integration, $\vec{v}(0°), \vec{v}(1°), \vec{v}(2°), \vec{v}(3°), \ldots$
are subject to the following condition and the obtained figure will be a polyhedron.

$\vec{v}(1°) - \vec{v}(0°) = \vec{a}_{180°}$ $\vec{v}(0°), \vec{v}(1°), \vec{v}(2°), \vec{v}(3°), \ldots$ $\vec{v}(2°) - \vec{v}(1°) = \vec{a}_{181°}$

$\vec{v}(3°) - \vec{v}(2°) = \vec{a}_{182°}$

If we take the limit, there is a circle.

- When the velocity vector draws a circle, the orbit will be a quadratic curve.
- You can verify that using KSEG.



We could explain the elliptic orbit without solving the differential equation.

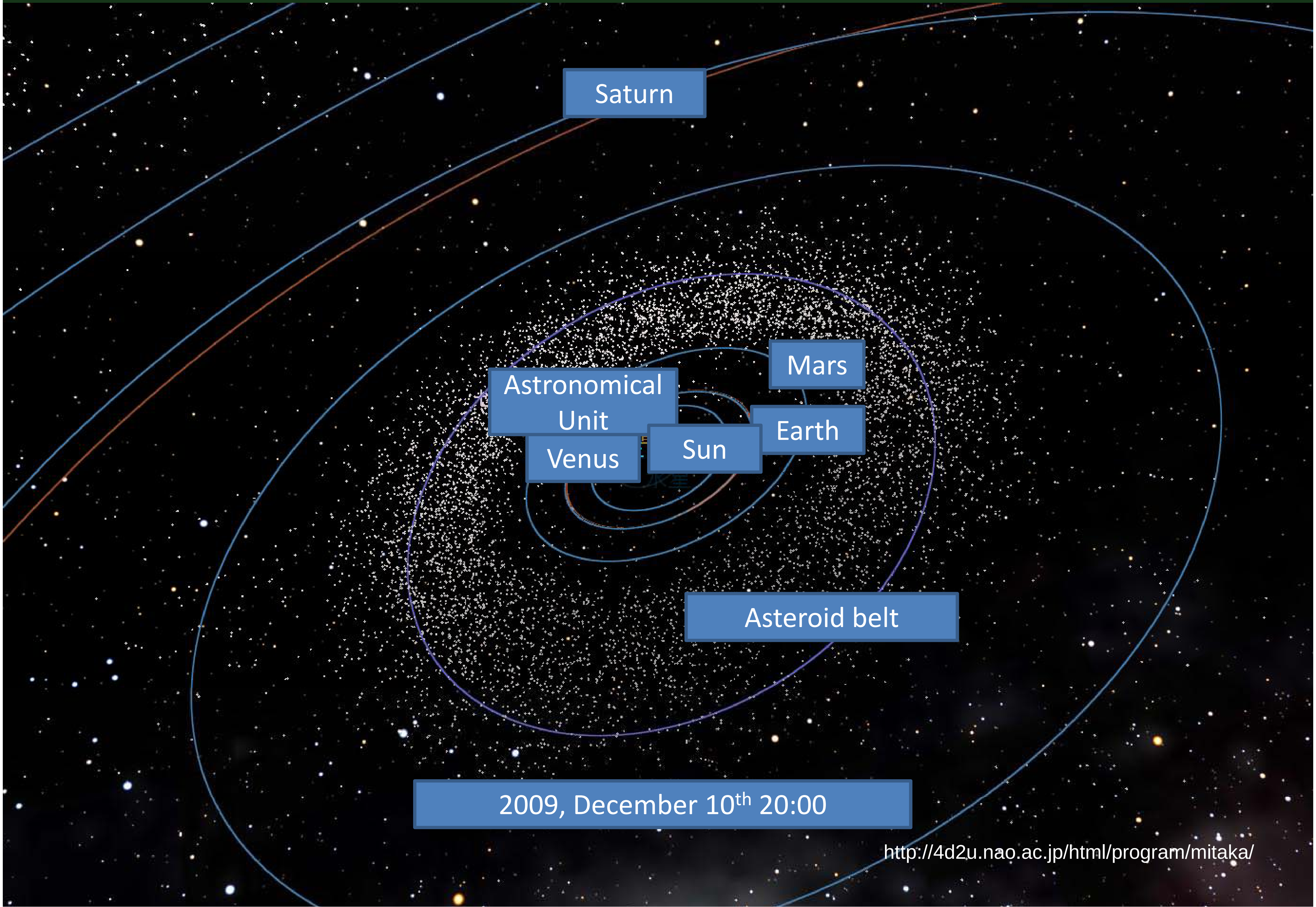
To solve differential equation numerically, you can write some BASIC codes.

```
DEF f(x,y) = − x/SQR((x^2+y^2)^3)
DEF g(x,y) = − y/SQR((x^2+y^2)^3)
FOR i=5 TO 19
  LET x=0.5
  LET y=0
  LET u=0
  LET v=0.1*i
  LET h=0.000001
  LET t=0
  PLOT POINTS: x,y
  DO WHILE (t<=0.8 OR x<0 or  y<=0)
    LET t=t+h
    LET uu=u
    LET vv=v
    LET u=u+f(x,y)*h
    LET v=v+g(x,y)*h
    LET x=x+uu*h
    LET y=y+vv*h
    PLOT POINTS: x,y
  LOOP
NEXT i
END
```

Today’s lecture so far

- To understand the orbits of planets, a lot of kinds of mathematics was necessary.
- Kepler’s law was stated, based on the knowledge of ellipses.
- Newton created the new mathematics and described mechanics.

Let’s review its form with Mitaka.


Saturn
Mars
Astronomical Unit
Earth
Sun
Venus
Asteroid belt
2009, December 10th 20:00
http://4d2u.nao.ac.jp/html/program/mitaka/

- If you are a student at natural sciences, you will take courses dealing with differential equations at the 2nd year, mathematical sciences 2 and mathematical sciences 4.

- Newton not only invented the concept of differentiation, but also knew the fundamental theorem of differential calculus, which says that differentiation and integration are the inverse manipulations.

- Vector analyses, dealt with in mathematical sciences 1 and mathematical sciences 3, are also related to this topic.

What kind of research am I engaged in?

things don’t go well when you try to represent the solutions of differential equations using only the well-known functions.

But you may be able to grasp the characteristic features of the solutions of differential equations without knowing the explicit form of their solutions, or even without knowing how to solve them.

Since my research is about the things shown above, in the next lecture I will describe related topics.

- So far, we have seen if a particle feel a central force whose magnitude is inversely proportional to the distance between the particle in question and the center, the orbit of the particle will be an ellipse (a conic curve, namely an ellipse, a parabola, or a hyperbola).

- The problem of two stars moving under mutual gravitational force will be, with slight modification, reduced to the equation of motion under central force.

- From the law of action and reaction, the center of gravity of two stars will be an inertial motion, and we can take a coordinate with the center of gravity as the origin.

- You can solve the differential motion, we get the following results.

BASIC codes

- In brief, the solution of two-body problem is an ellipse (a conic curve) whose center is the center of gravity.
- Newton seems to have noticed this fact.
- Then, Newton himself, while remarking the importance of investigating three-body problem, said that the problem wouldn't be solved with already known methods or knowledge.
- As a mathematical problem, even the three-body problem turned out to be so difficult.

BASIC codes

- We could explain the elliptic orbit of planets because the mass of planets are so small compared with the mass of the Sun that the force between planets are negligible.
- The role of gravitational force between planets are to deflect the orbits of planets from ellipses.
- Such gaps have become observable thanks to reflecting telescopes and photo plates.
- Leading to the discovery of Neptune in 1846.
- Both optical aberration (in 1727) and parallax (in 1838) are observed, and the theory of heliocentric system is verified.

Reference: Principia
http://en.wikipedia.org/wiki/Philosophia_Naturalis_Principia_Mathematica

Reference:
Matsumoto makoto,
http://mathsoc.jp/publication/tushin/0401/matsumoto41.pdf

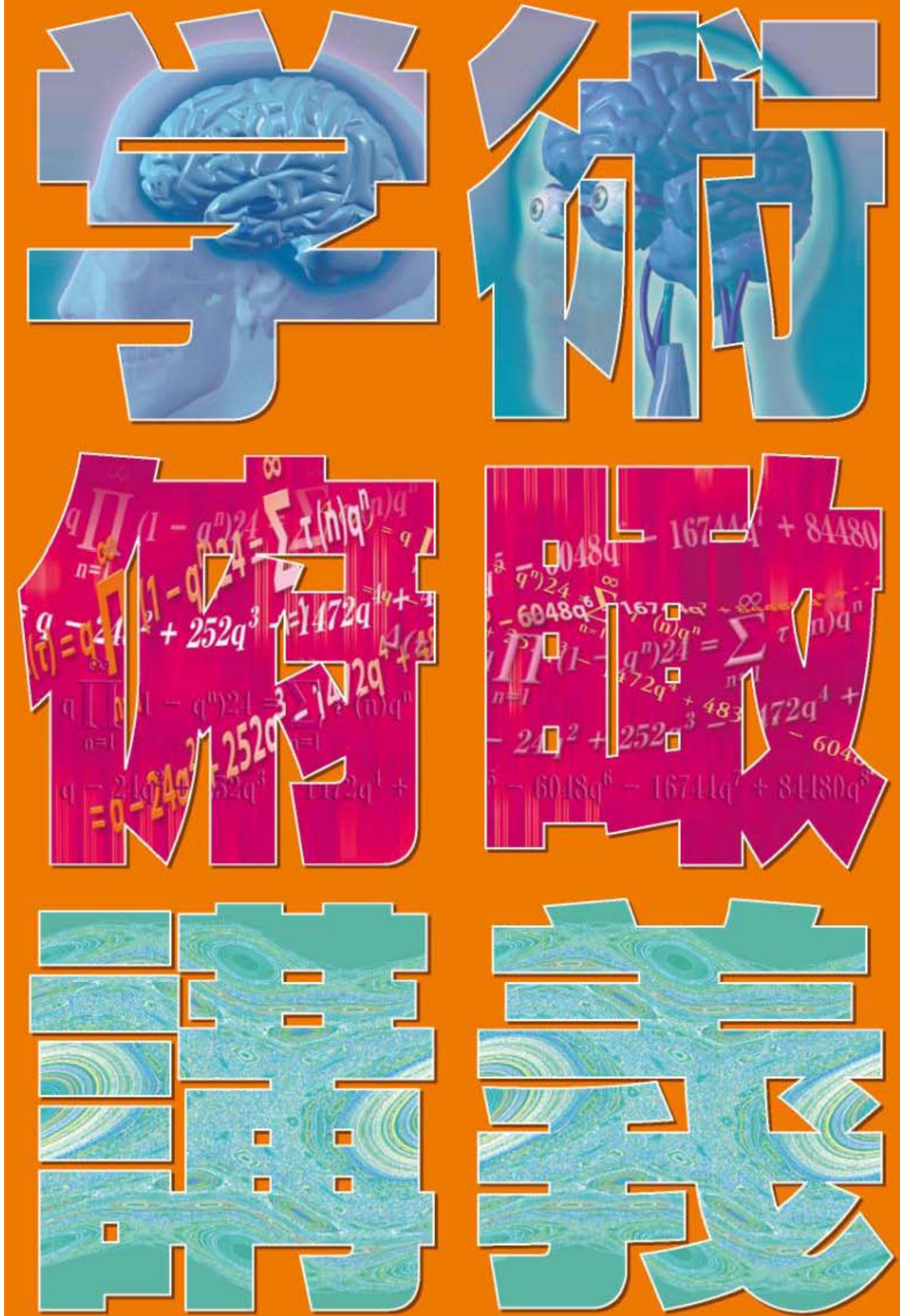
Reference:
The MacTutor History of Mathematics archive
(good articles on the history of mathematics)
http://www-history.mcs.st-andrews.ac.uk/history/index.html

Tycho Brahe, Johannes Kepler, Sir Isaac Newton, Gottfried Wilhelm von Leibniz, Henri Poincare
MathematicalPhysics/Orbits/

Reference:
Mitaka (free software) is downloadable for free from
http://4d2u.nao.ac.jp/html/program/mitaka/
the website of National Astronomical Observatory of Japan.

Reference:
the following software can be obtained for free:

KSEG http://www.mit.edu/~ibaran/kseg.html
Jisshin-BASIC http://hp.vector.co.jp/authors/VA008683/



教科書にはのっていない数学のお話

主題科目／テーマ講義　２単位　１、２年生対象

数学を創る－数学者達の挑戦－

コーディネータ・ナビゲータ：岡本和夫（理学部）

数学はどうやって創られたか　岡本和夫（理学部）、室田一雄（工学部）

第1回	10/8	数学はどうやって創られたか

ことばを創り、世界を創る　斎藤毅（理学部）

第2回	10/15	Mathematics "On Campus"
第3回	10/22	数の体系を創る
第4回	10/29	数と図形の共進化

脳と情報の数学を創る　甘利俊一（理化学研究所）

第5回	11/5	情報の仕組み：驚き、確率、幾何学
第6回	11/12	脳の仕組み：脳内情報の表現、記憶、学習の数理

目の錯覚の数学を創る　新井仁之（理学部）

第7回	11/19	数学で探る錯視の世界
第8回	11/26	脳の中のウェーブレット
第9回	12/3	錯視が創る新たな数学―ウェーブレットからフレームレットへ―

形を理解するための数学を創る　坪井俊（理学部）

第10回	12/10	惑星の軌道を理解する
第11回	12/17	多面体の形と曲面の上の軌道の形
第12回	1/14	形の見分け方と数学の視点

文化と数学　岡本和夫（理学部）

第13回	1/21	文化と数学

駒場キャンパス 18号館ホール 木曜日 5時限（16:20-17:50）

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