東京大学
THE UNIVERSITY OF TOKYO

11/11/2010
Global Focus on Knowledge

The World of Diverse Matter
— The Distant Journey from Space to Earth

The Diversity of Materials Borne from the Collective Actions of Atoms, Electrons, and Molecules

6th Lecture

Quantum World
Nanoscience, Superconductivity, Superfluidity

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The University of Tokyo

Today's Talk

- About Quantum Mechanics
  - Quantum Interference, Tunnel Effect
- Nano-science
  - Mesoscopic Physics
  - Scanning Probe Microscope
- Macroscopic Quantum Phenomena
  - Superfluidity
  - Bose Condensation
  - Superconductivity
- Summary

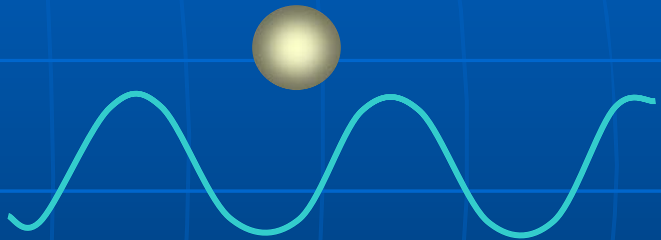
Brief Introduction to Quantum Mechanics

Quantum Mechanics

Theoretical framework to describe the workings of the micro world

Structure of atoms/molecules
Behavior of electrons in solids
Light and Substances

Light: both wave-like and particle-like
Electron: both particle-like and wave-like



The particle/wave dichotomy only reflects the fact that we have no suitable expressions when we try to describe quantum mechanical behaviors in terms of everyday (classical mechanical) words.

Particle nature: discrete, countable
Wave nature: superposition, interference

Quantum Mechanical Particles also Exhibits Wave-like Behavior

de Broglie Wavelength $\lambda = \dfrac{h}{p}$ ← Momentum

The de Broglie wavelength of an electron that is accelerated at 100V

$$E = \frac{p^2}{2m} \quad \Rightarrow \quad p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.62\times10^{-34}}{\sqrt{2\times0.91\times10^{-30}\times100\times1.6\times10^{-19}}}$$

$$= 1.23\times10^{-10}\,\text{m} \qquad \lambda = 0.12\,\text{nm}$$

The de Broglie wavelength of a classical particle (*e.g.* a tennis ball) is *extreeeeemely* short

Wave Function

The state of a quantum mechanical particle is described by a wave function $\psi(x, y, z, t)$

The probability of finding the particle at a position (x,y,z) at time t is given by $\left|\psi(x, y, z, t)\right|^2$

The time evolution of a wave function obeys Schrödinger's equation

$$i\hbar\frac{d}{dt}\psi(x,y,z,t)=\left(-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(r)\right)\psi(x,y,z,t)$$

This is a linear equation => Superposition Principle

$$\psi_1, \quad \psi_2 \quad \Rightarrow \quad \psi_1+\psi_2$$

=>Quantum Interference

The Role of Measurements in Quantum Mechanics

Quantum mechanics provides a probabilistic distribution of the outcome of the measurements of a physical quality when they are repeated in the "same situation". However, it does not generally give a definite value for an individual measurement.

Immediately after a measurement, the state takes one of the eigenstates of the physical observable.
=> "Collapse of state"

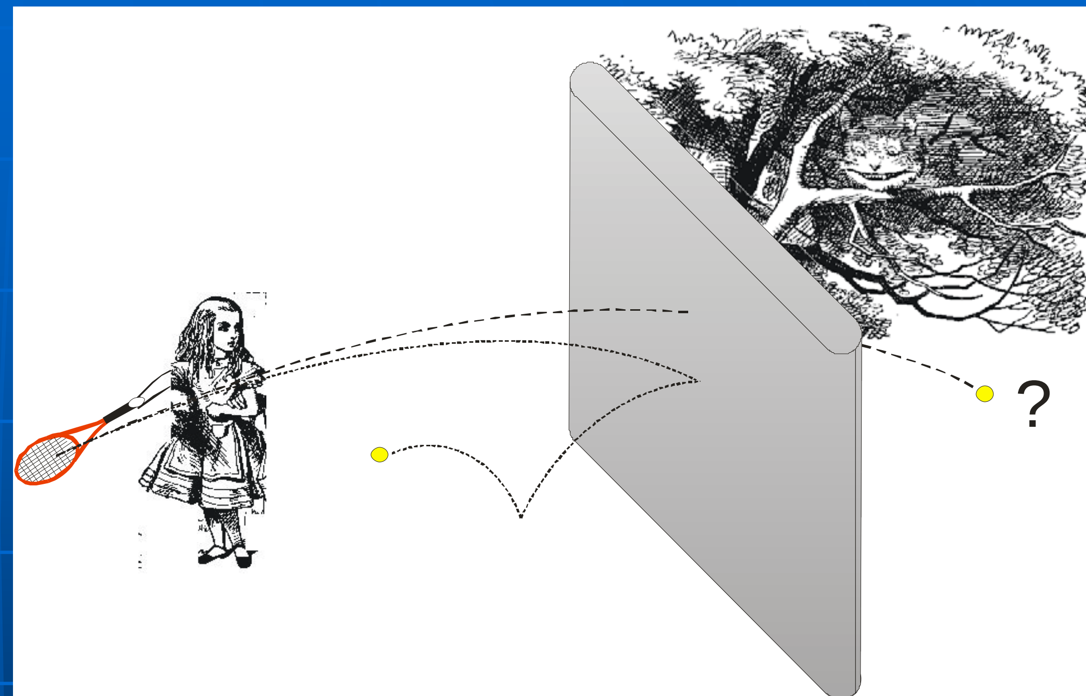
The unitary time evolution is described by Schrödinger's equation, whereas measurements give rise to collapse of the state.
The standard interpretation of quantum mechanics
(Copenhagen Interpretation)

Interpretation problem, observation problem of quantum mechanics
Many-Worlds Interpretation

Characteristic Quantum Mechanical Phenomena

Tunnel Effect

- Quantum mechanical particles can go through a potential wall they cannot penetrate according to classical dynamics

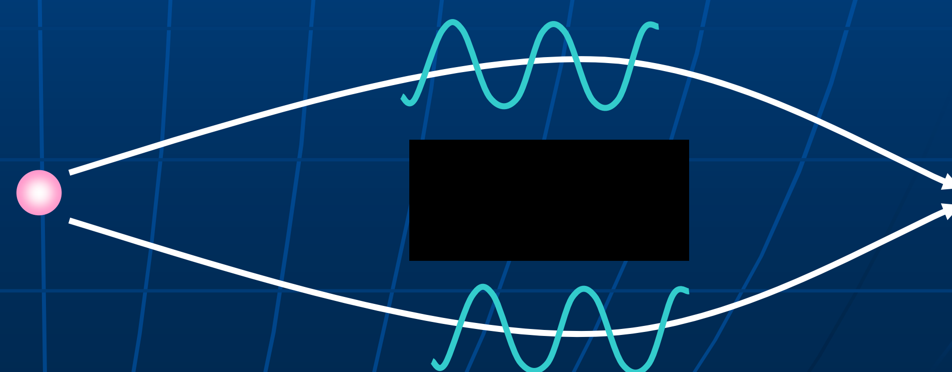


Yasuhiro Iye 2006 “Alice’s quantum mechanics”(In Japanese) Maruzen‡

Quantum Interference Effect

- Superposition of states that pass through different paths

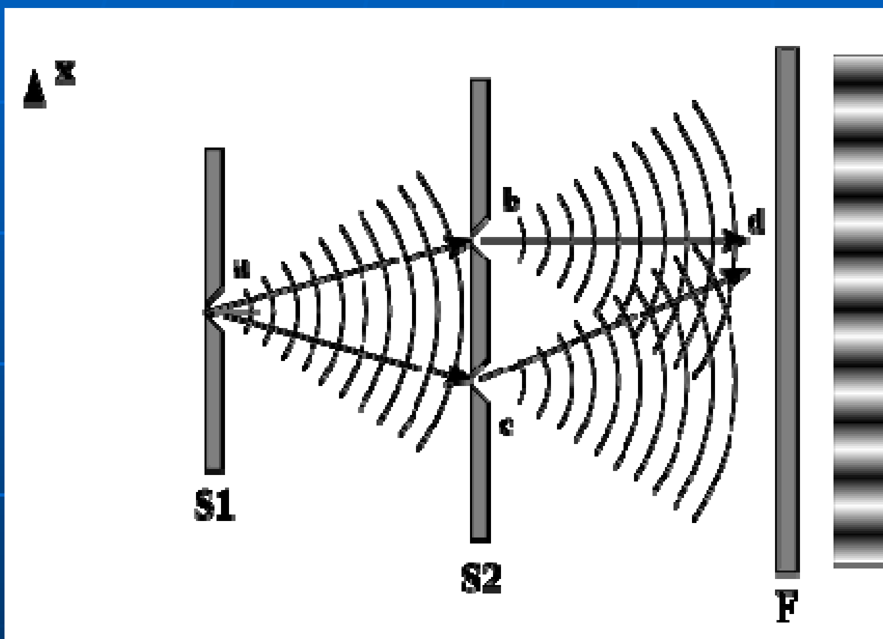
=> Quantum Interference



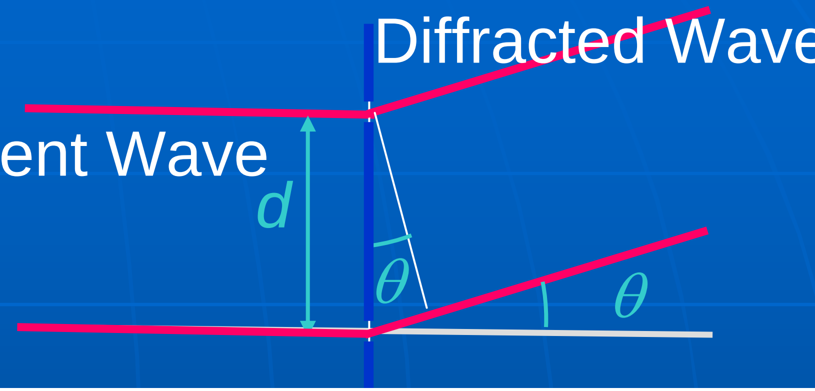
Quantum Interference

Interference of Light Waves

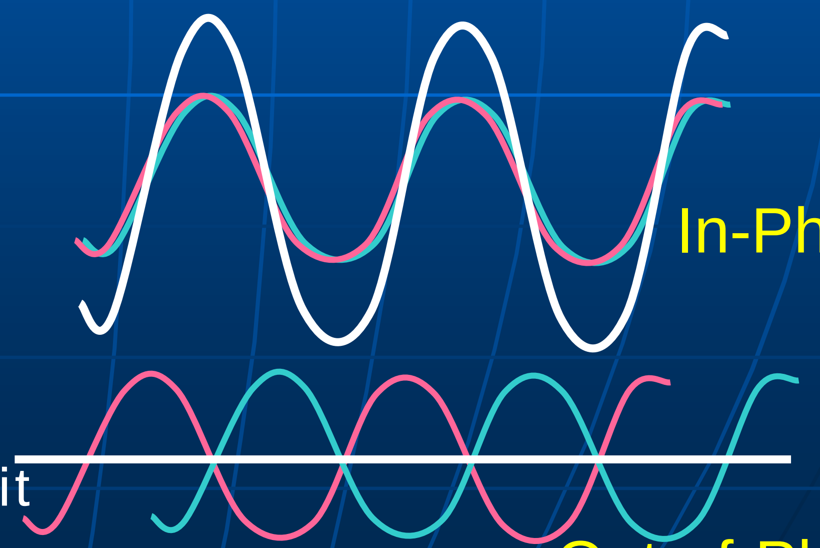
Young's double-slit experiment (1805)



‡ http://en.wikipedia.org/wiki/File:Ebohr1.svg

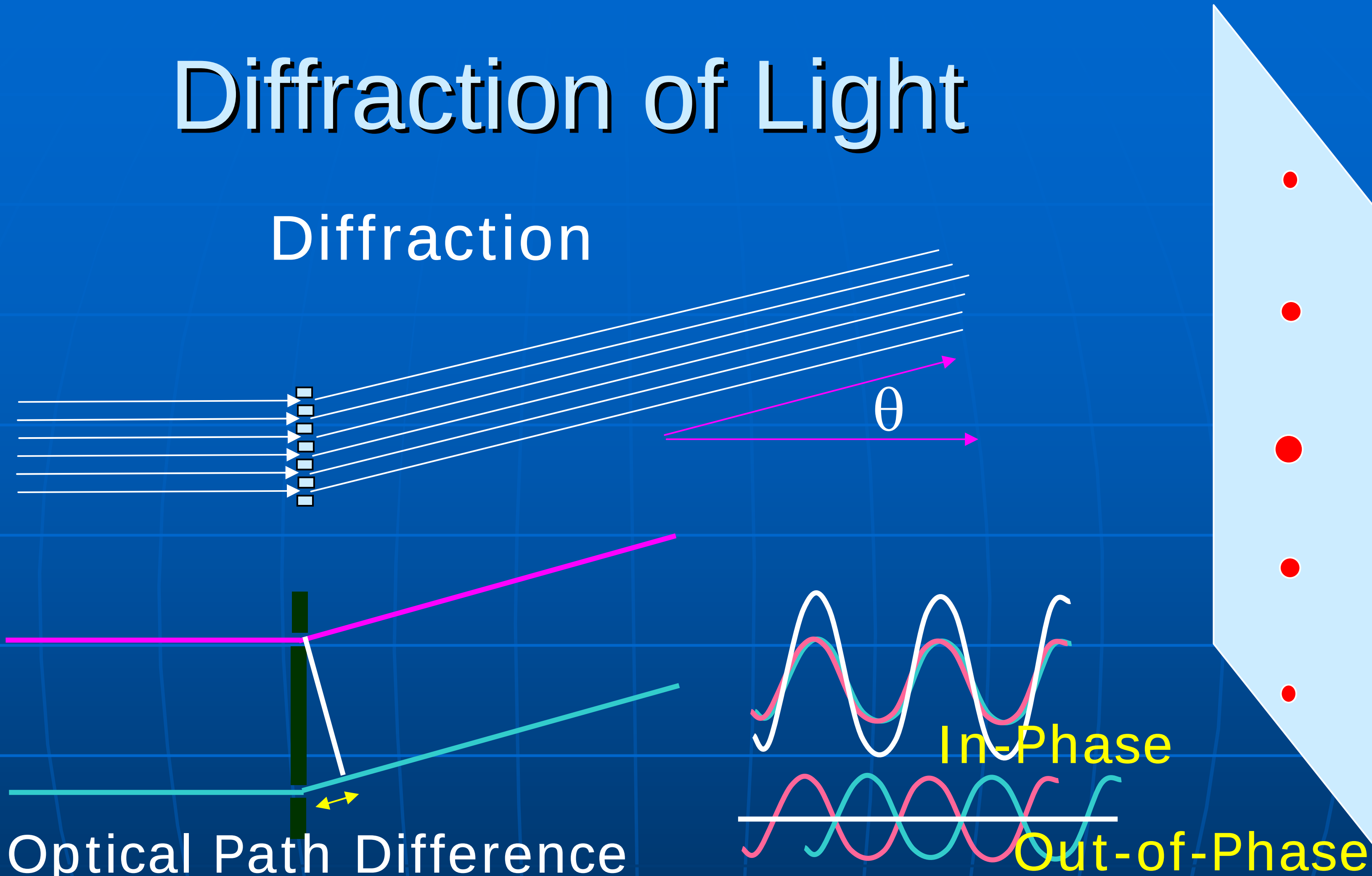


$$d\sin\theta = \begin{cases} n\lambda & \text{enhance} \\ \left(n+\frac{1}{2}\right)\lambda & \text{cancel} \end{cases}$$



Interference of waves
that passed through a double slit

Diffraction of Light

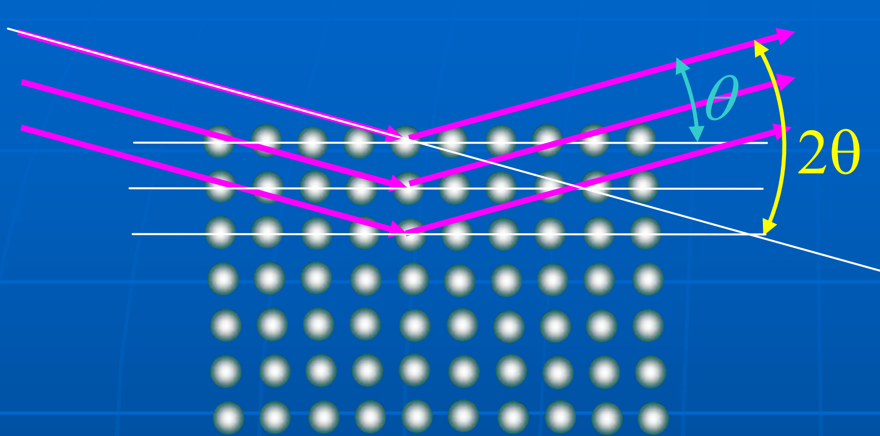


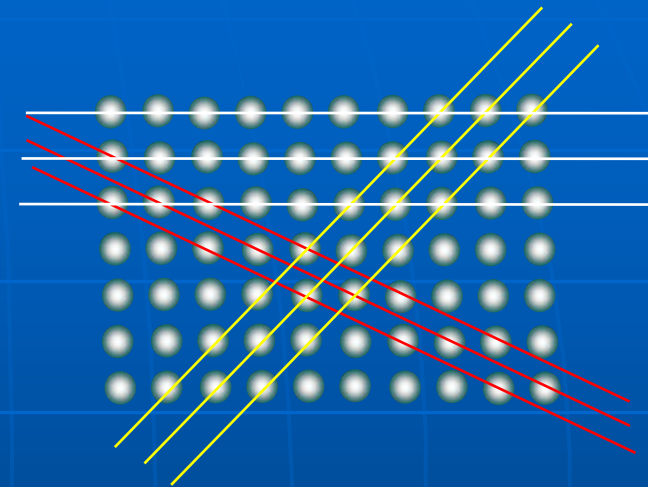
$$d \sin\theta = n\lambda$$

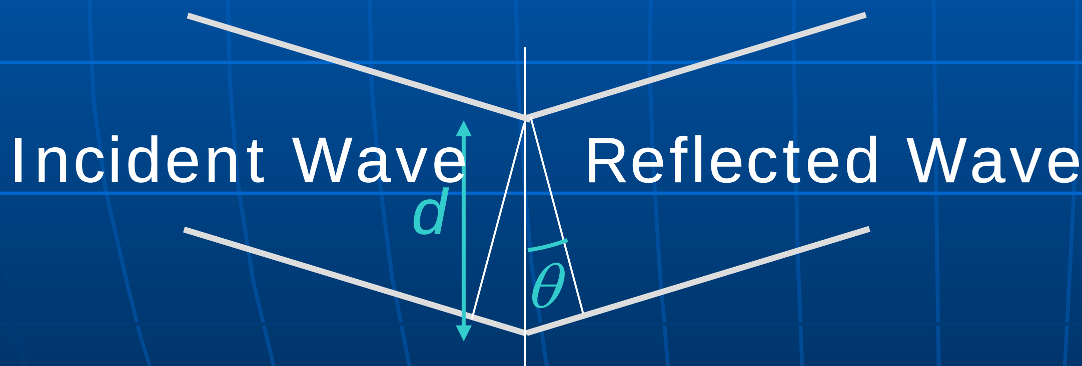
Inter-atomic Spacing ~0.3nm~Wavelength of X-Rays

Diffraction experiment can be also done with electron beams and neutron beams.

Crystal Structure Analysis





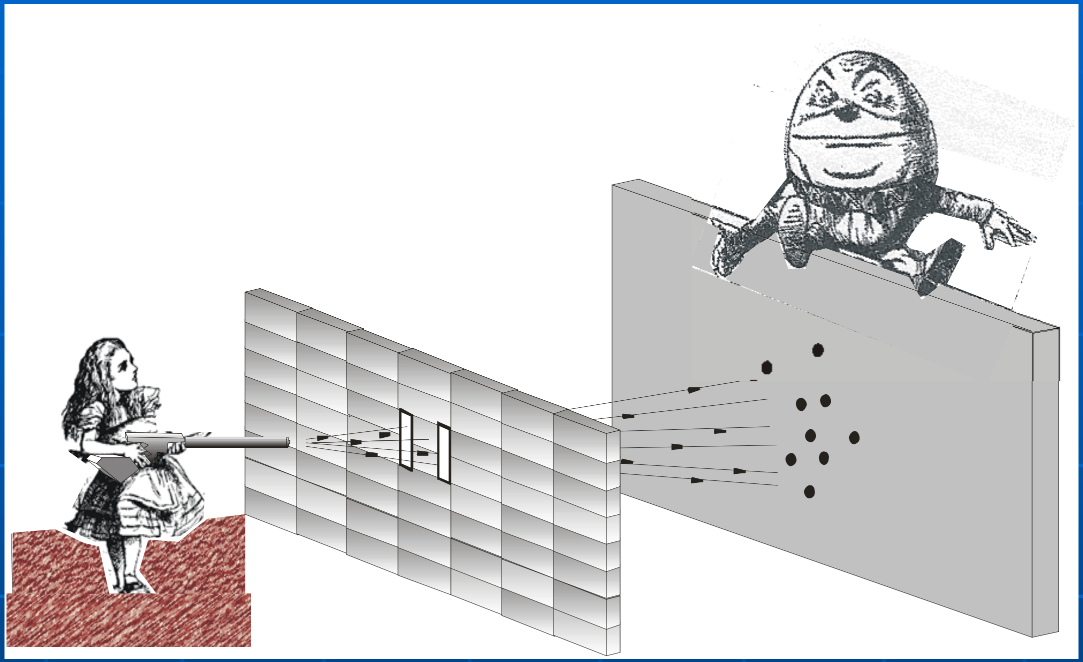


$$2d\sin\theta = n\lambda$$

Bragg Condition

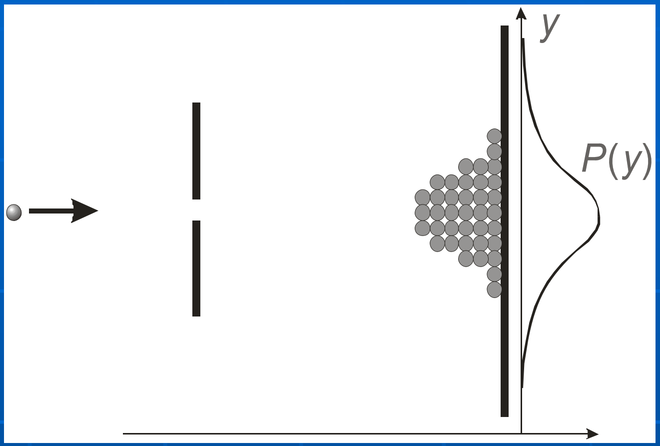
Four-Axes X-ray Diffractometer

The Case of Classical Particles

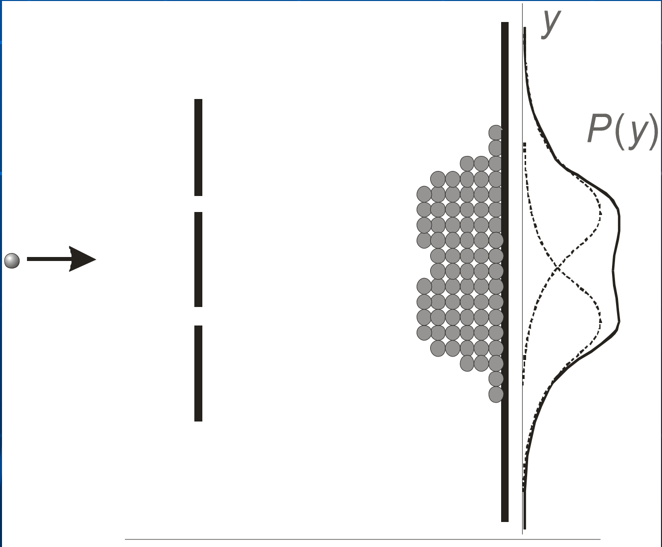


Yasuhiro Iye 2006 “Alice’s quantum mechanics”(In Japanese) Maruzen‡

1 slit



2 slits

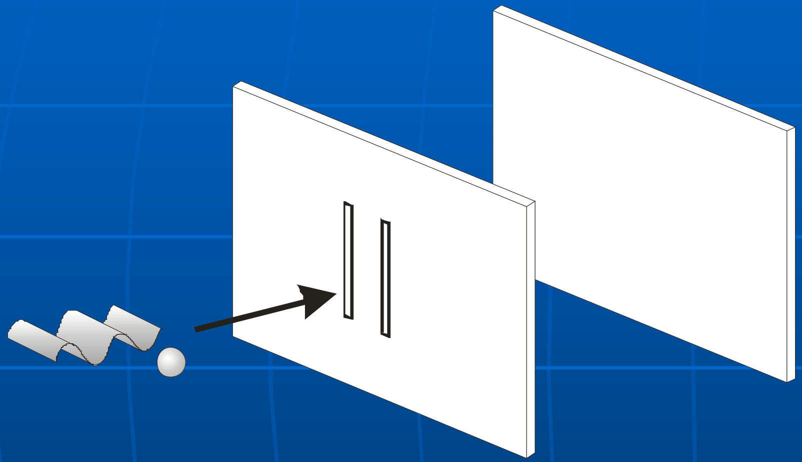


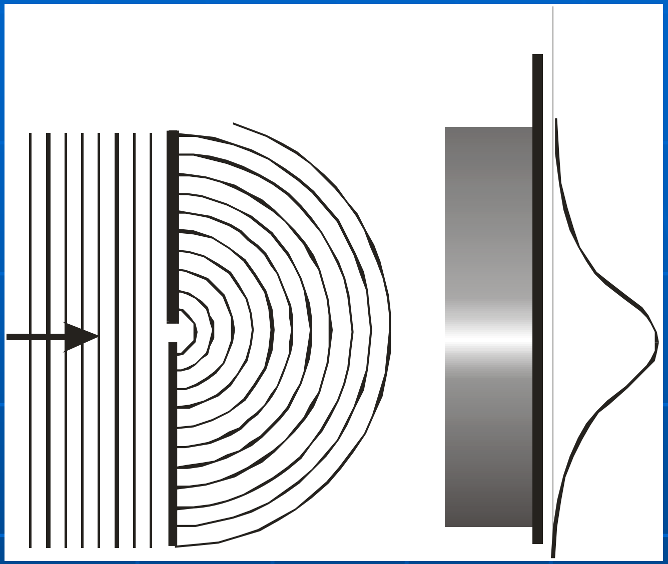
Probability of a bullet hitting a particular spot
＝Probability of reaching the spot via the right slit
＋Probability of reaching the spot via the left slit

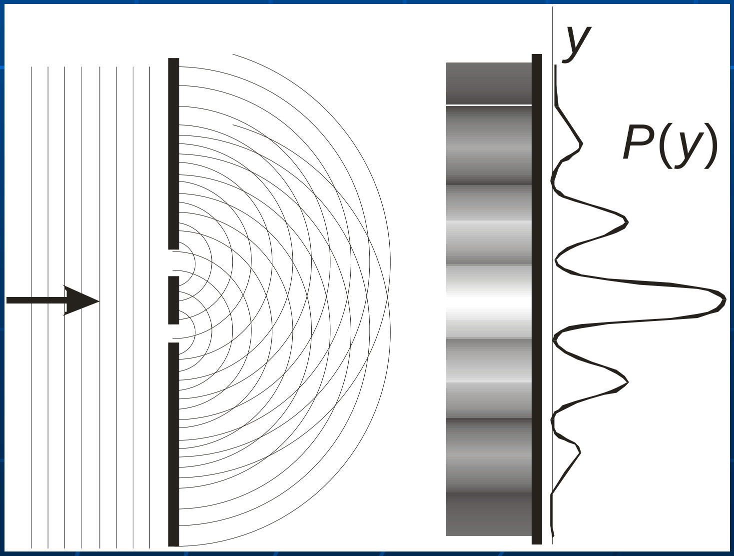
$$P_{\text{total}}(y) = P_{\text{R}}(y) + P_{\text{L}}(y)$$

Summation of Probability

The Case of Waves







Summation of
Wave Amplitudes

Interference Pattern

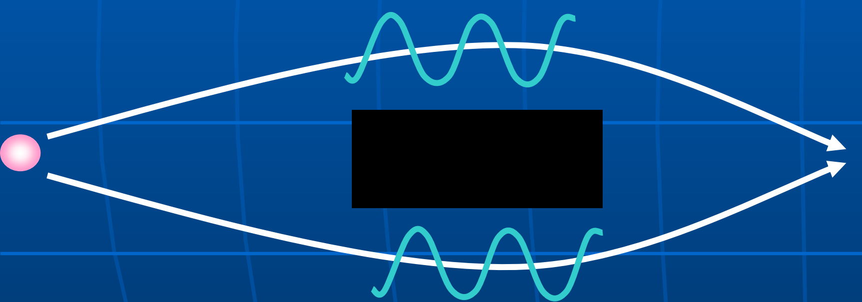
The Case of
Quantum Mechanical Particles

$$\Psi_{\text{total}} = \Psi_R + \Psi_L$$

Total Wave Function

= Wave Function passing through right-hand slit

\+ Wave Function passing through left-hand slit

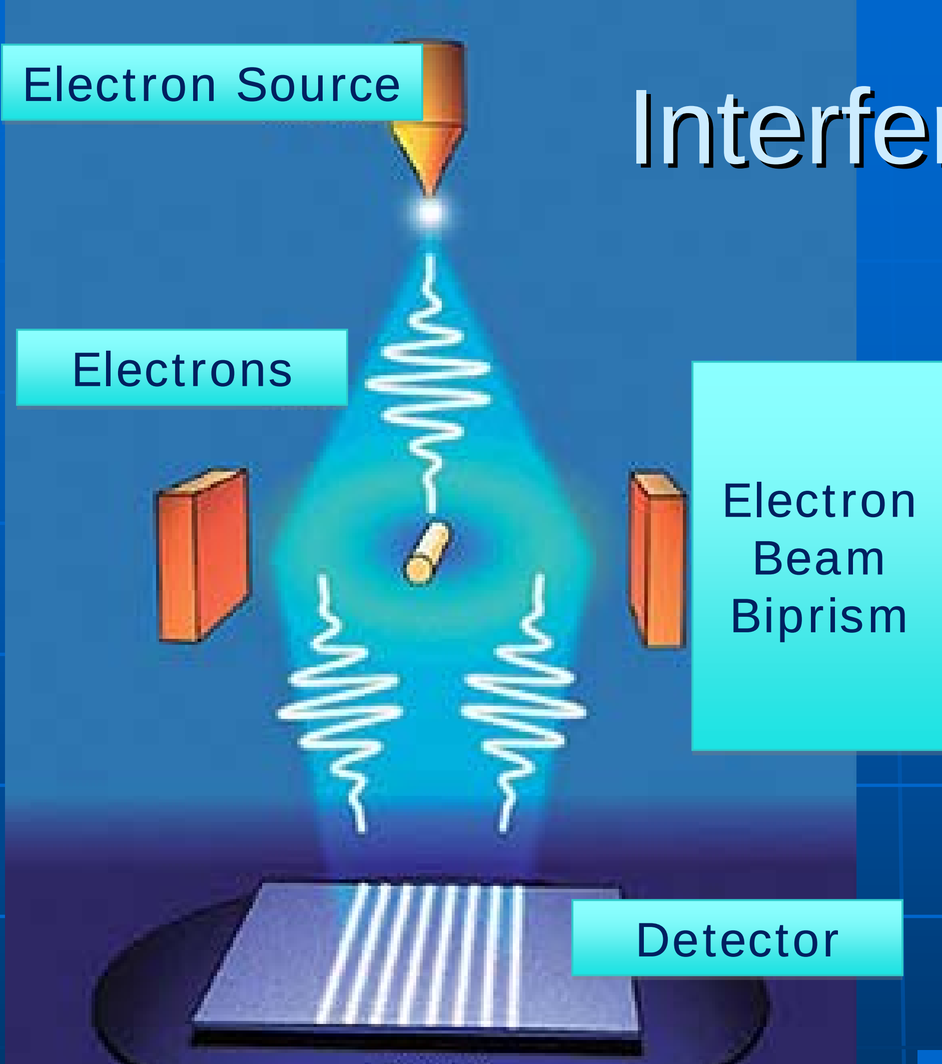


Probability = |Wave Function|^2

$$\left|\Psi_{\text{total}}\right|^2 = \left|\Psi_R + \Psi_L\right|^2$$
$$= \left|\Psi_R\right|^2 + \left|\Psi_L\right|^2 + \Psi_R^*\Psi_L + \Psi_R\Psi_L^*$$

Quantum Interference Terms

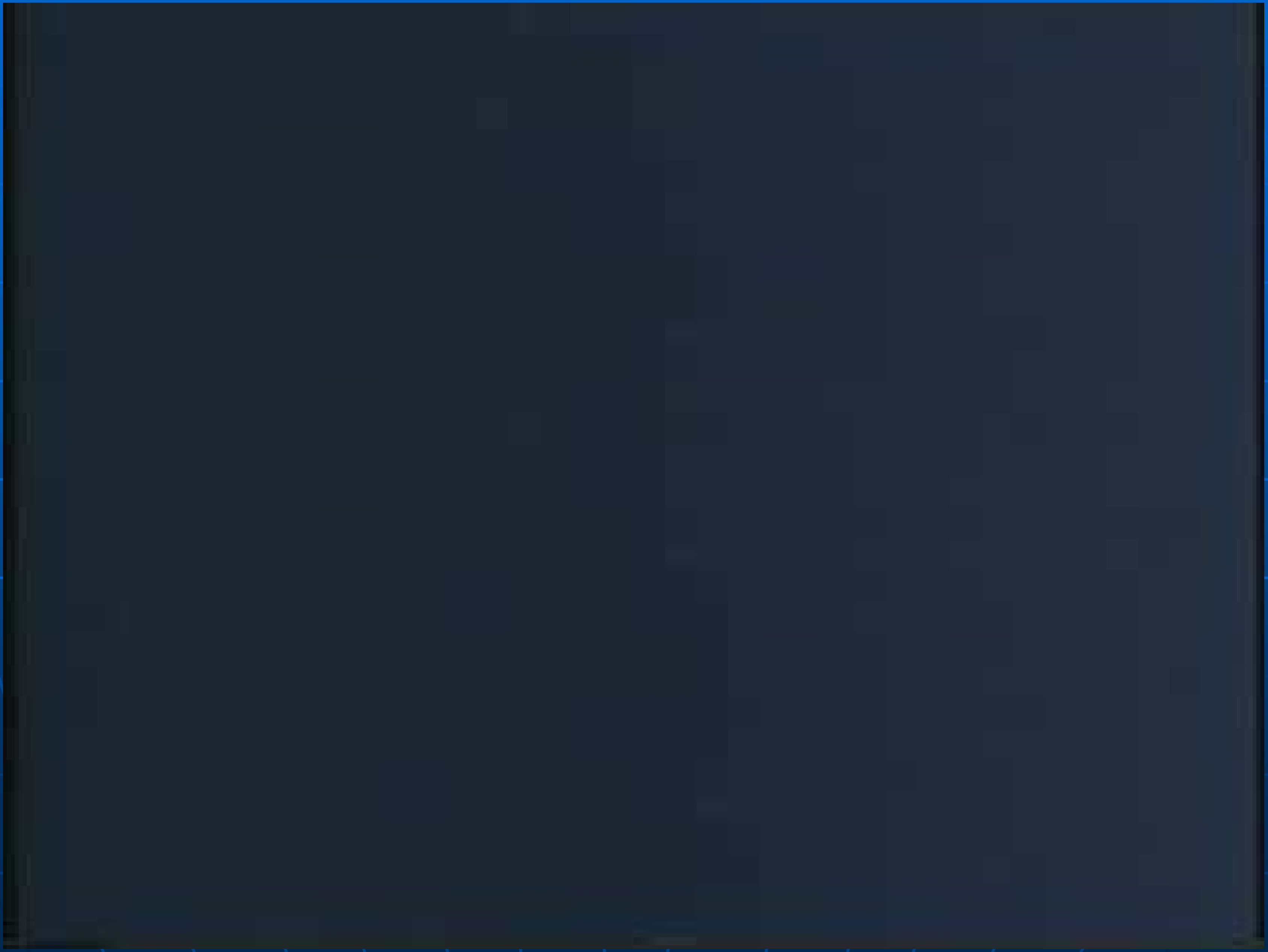
Interference of Electrons



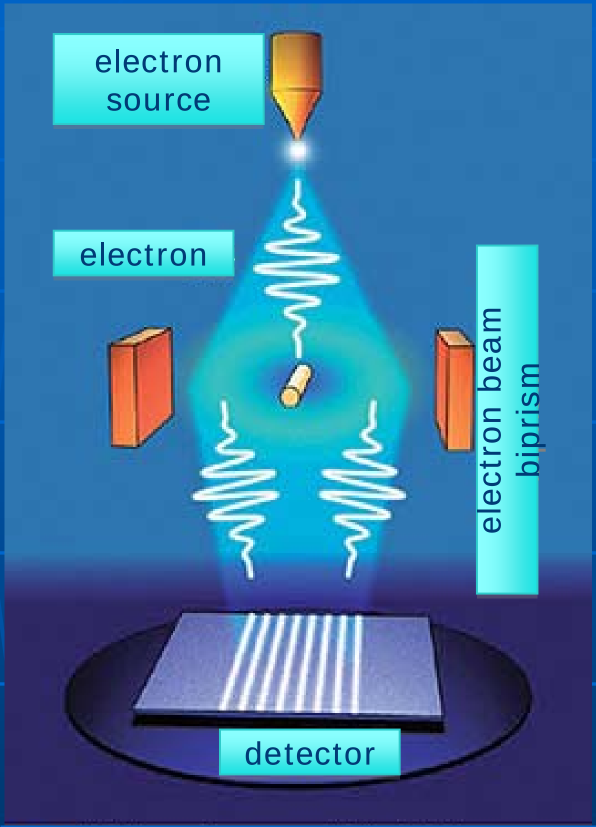
Dig. 3 Representation of Electron Two-Slit Experiment
In this experiment an electron beam bi-prism is used in place of a slit. This type of experiment is technically difficult and at one time was thought of as a thought experiment that could only be done in your head.

Dr Akira Tonomura (Hitachi Advanced Research Laboratory) ‡

Double-Slit Experiment with Electrons (Dr Akira Tonomura)



Interference of Electrons



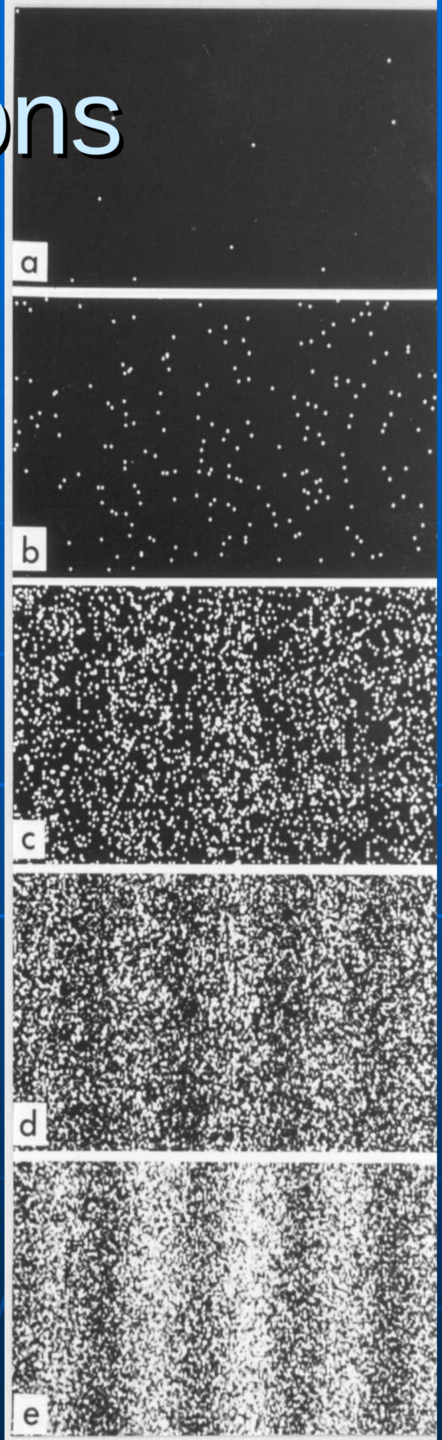
Dr Akira Tonomura (Hitachi Advanced Research Laboratory)

Electrons reach the screen one-by-one

Interference pattern emerges

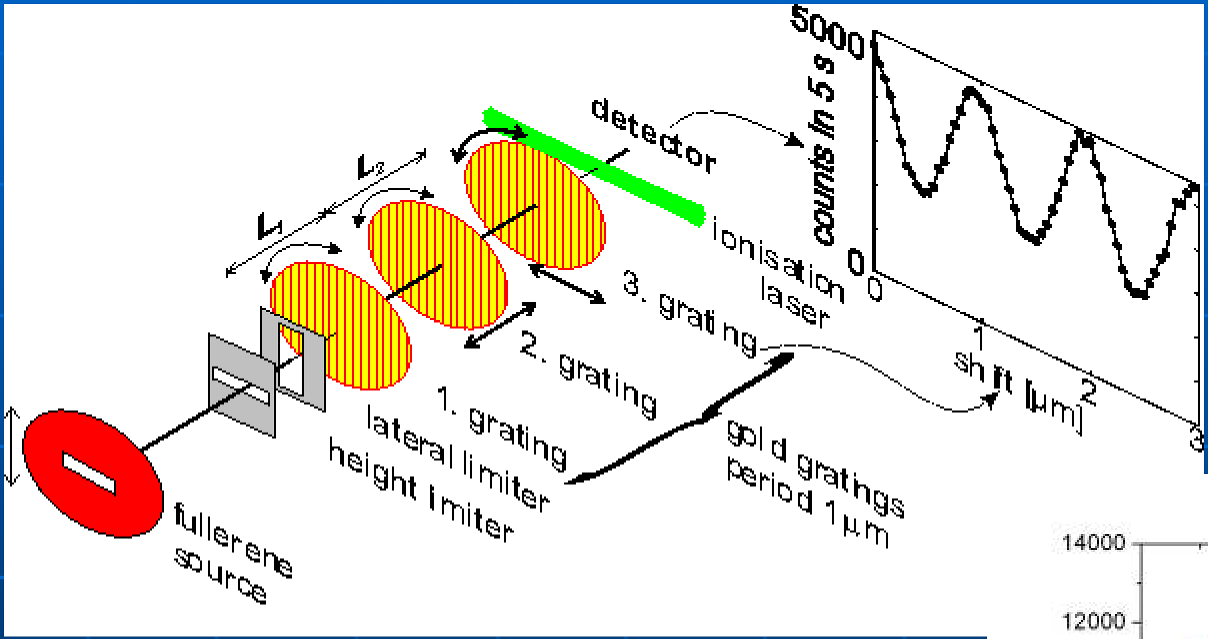
Unambiguous proof of the wave nature of electron.

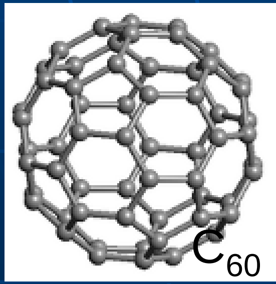
‡



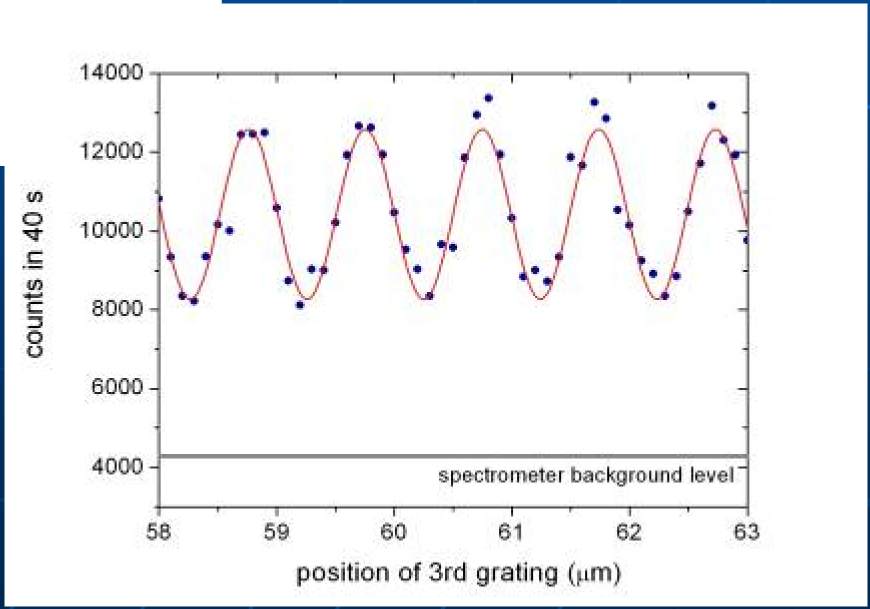
How Large an Object Can Still Interfere?

A.Zeilinger
Vienna University of Technology





C_{60} Fullerene molecule



Aharonov-Bohm (AB) Effect

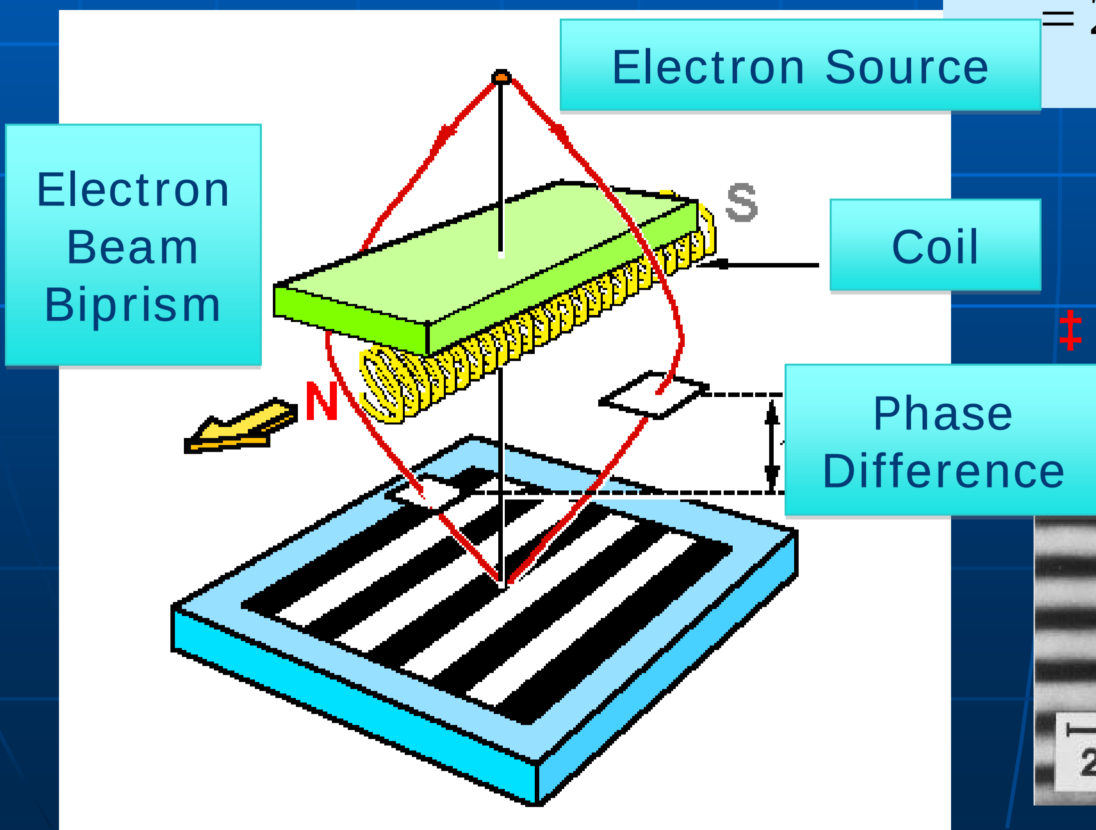
The phase of electron wave changes with magnetic field (more precisely, with vector potential)

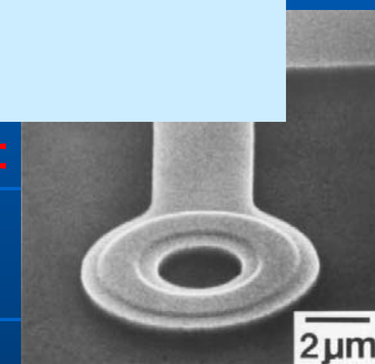
$$\psi \Rightarrow \psi e^{i\frac{e}{\hbar}\int \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}}$$

$$\Delta\theta = \frac{\hbar}{e}\int_{L}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} - \frac{\hbar}{e}\int_{R}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$

$$= \frac{\hbar}{e}\oint_{loop}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{\hbar}{e}\int \mathbf{B}(\mathbf{r})\cdot d\mathbf{S}$$

$$= 2\pi\frac{\phi}{\phi_0} \qquad \phi_0 = \frac{h}{e}$$





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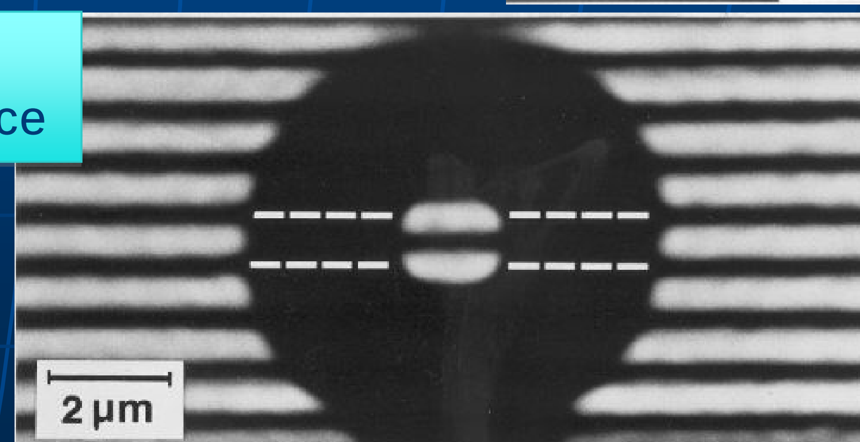


Mesoscopic System

Nanotechnology

Mesoscopic Physics

Micro-scale (Microscopic) ⟷ Macro-scale (Macroscopic)

Mesoscopic System

Intermediate range between micro and macro
(The prefix “meso” stands for intermidiate)

Physics characteristic to the systems whose scale is comparable to or smaller than the “characteristic length scale of the relevant physical phenomenon”
=> mesoscopic physics

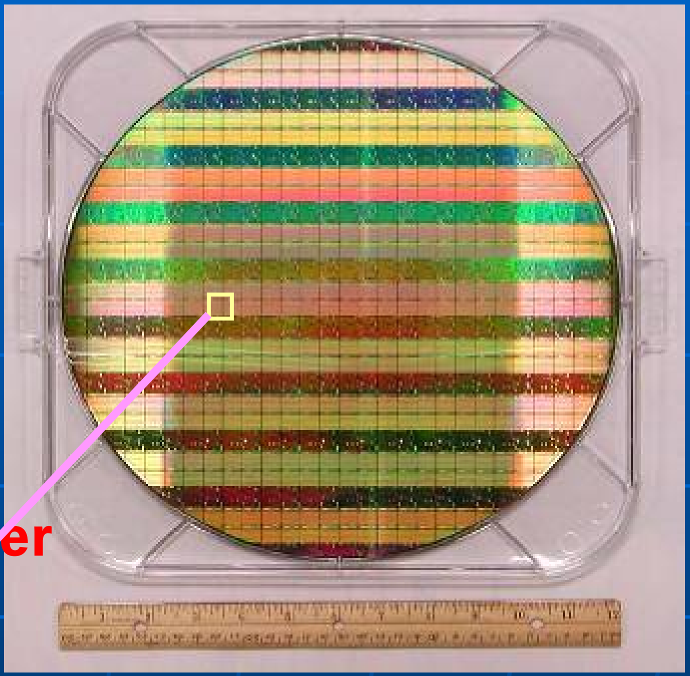
Examples of the “characteristic” length scale:
- Wavelength of an electron (Fermi wavelength)
- Mean free path of an electron (average distance an electron travels before it is scattered)
- Phase relaxation length of an electron

Miniaturisation of Semiconductor Devices

http://en.wikipedia.org/wiki/File:Replica-of-first-transistor.jpg

First Electrical Transistor
Bell Telephone Laboratories
(1946)

50 years



Silicon Wafer

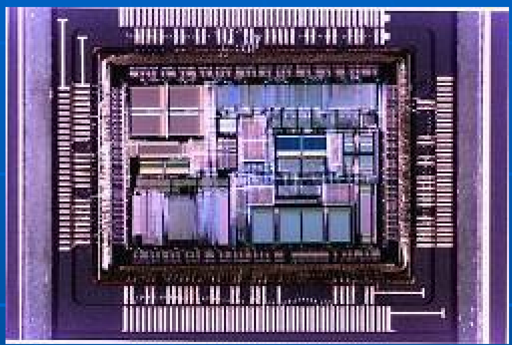
12 inches (30 cm)

Progress in Semiconductor
High-Tech Industry

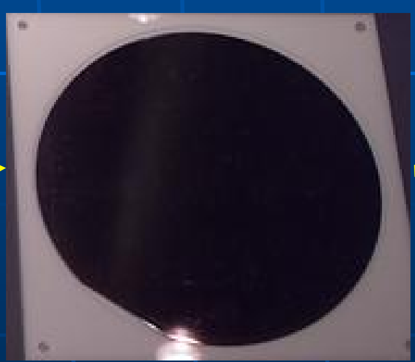
Silicon Single Crystal => Wafer => ULSI

ULSI

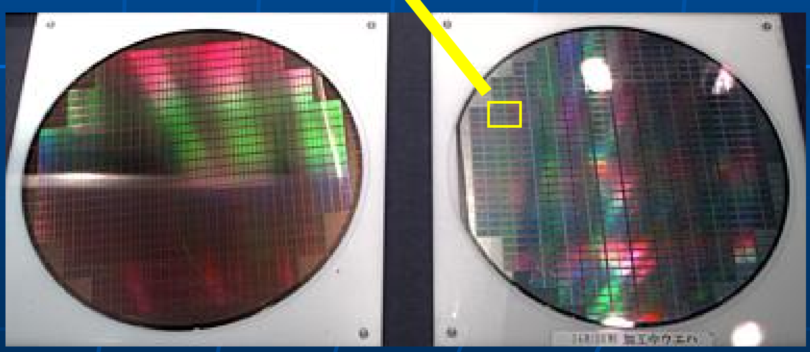
(Ultra Large-Scale Integrated Circuit)



Slicing and Polishing

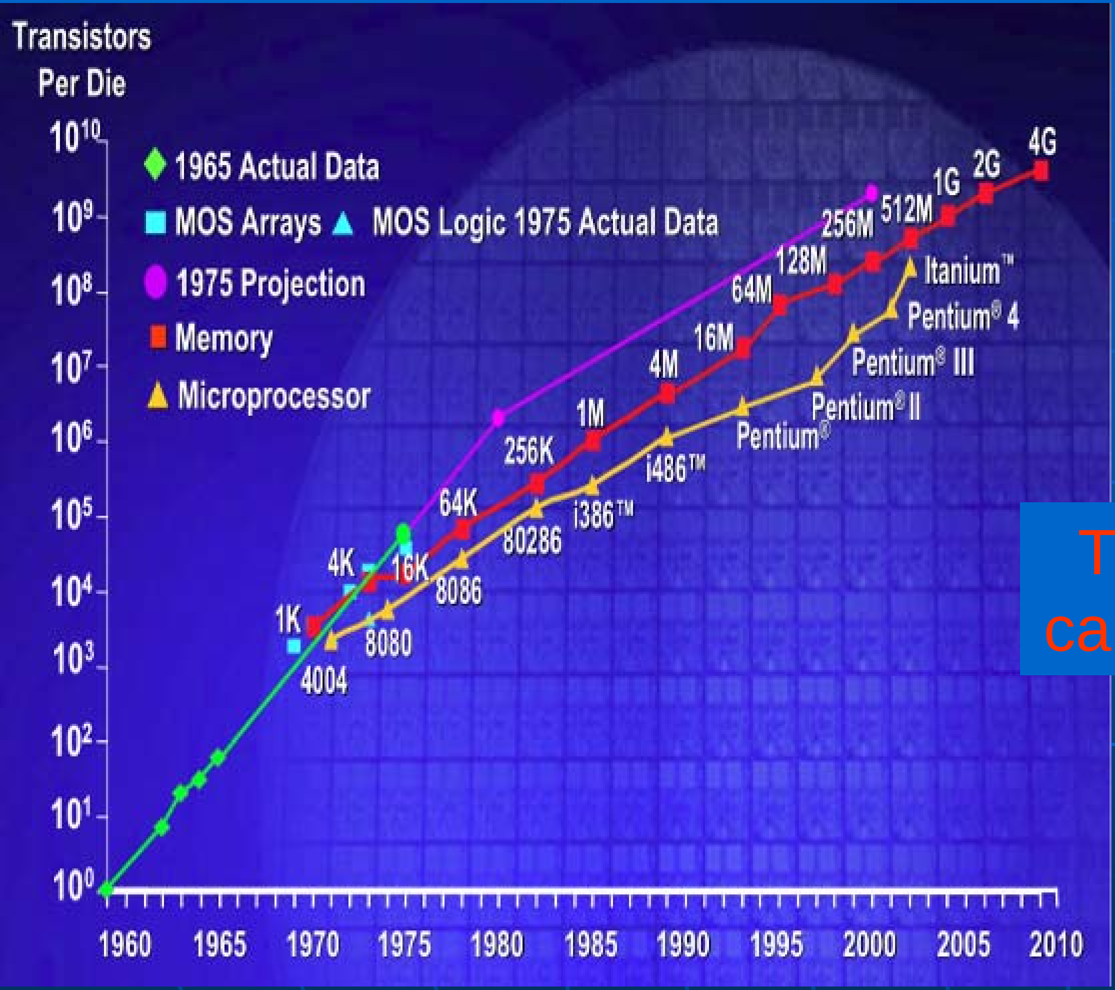


Wafer



Microfabrication

Moore’s Law



Gordon Moore (1965)
Co-founder of Intel

Typical example of the so-called self-fulfilling prediction

Gordon E.Moore “NO EXPONENTIAL IS FOREVER...”
‡Intel Copyright Permission Department

The degree of integration of LSI (Large Scale Integrated Circuit), namely the number of transistors that can be packed into a given area of semiconductor chip doubles every one-and-a-half or two years.

Nano-science / Nanotechnology

- Miniaturization technology ⇒ More than Moore

  It is not a simple scaling down
  - Problem associated with heat generation
  - Problem associated with quantum fluctuation

- Nano-science
  - Observing atoms, Manipulating atoms
  - Pursuit of new quantum effects in the nano-world

"There is plenty of room at the bottom."
(Richard Feynman)

Mesoscopic Physics

Aharonov-Bohm (AB) Effect

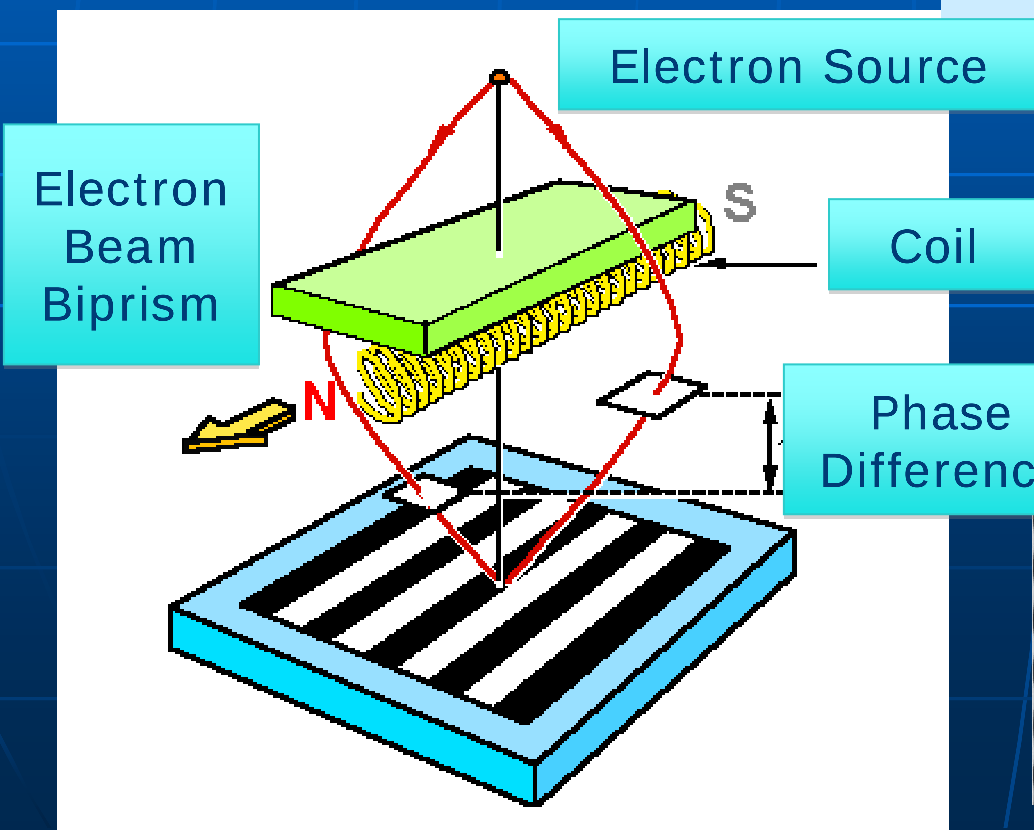
The phase of an electron wave changes according to a magnetic field (more precisely, with a vector po

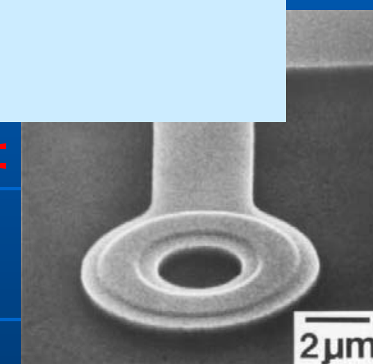
$$\psi \Rightarrow \psi e^{i\frac{e}{\hbar}\int \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}}$$

$$\Delta\theta = \frac{\hbar}{e}\int_{\mathrm{L}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} - \frac{\hbar}{e}\int_{\mathrm{R}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$

$$= \frac{\hbar}{e}\oint_{\mathrm{loop}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{\hbar}{e}\int \mathbf{B}(\mathbf{r})\cdot d\mathbf{S}$$

$$= 2\pi\frac{\phi}{\phi_0} \qquad \phi_0 = \frac{h}{e}$$





Dr Akira Tonomura
(Hitachi Advanced
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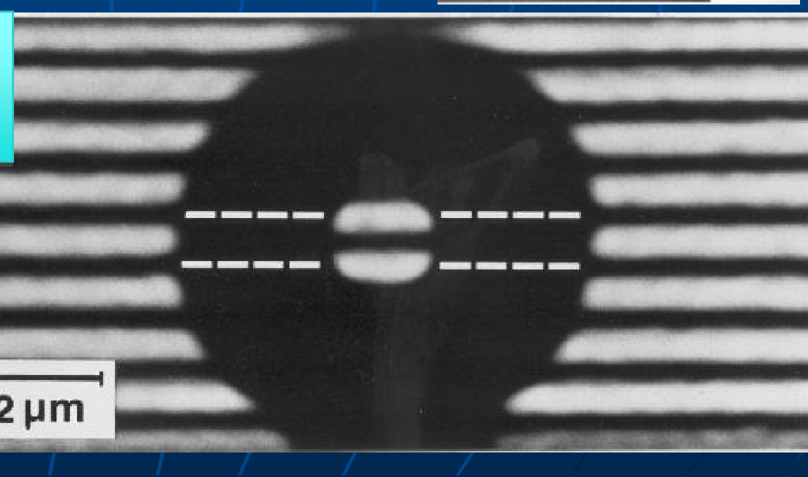
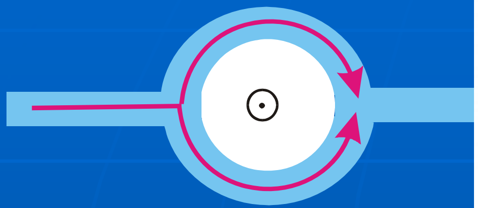
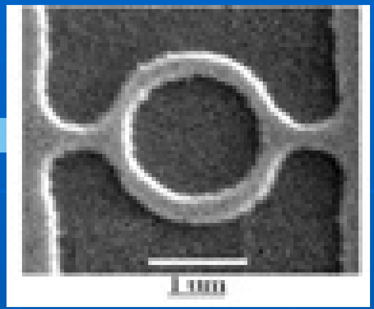


Aharonov-Bohm Oscillations

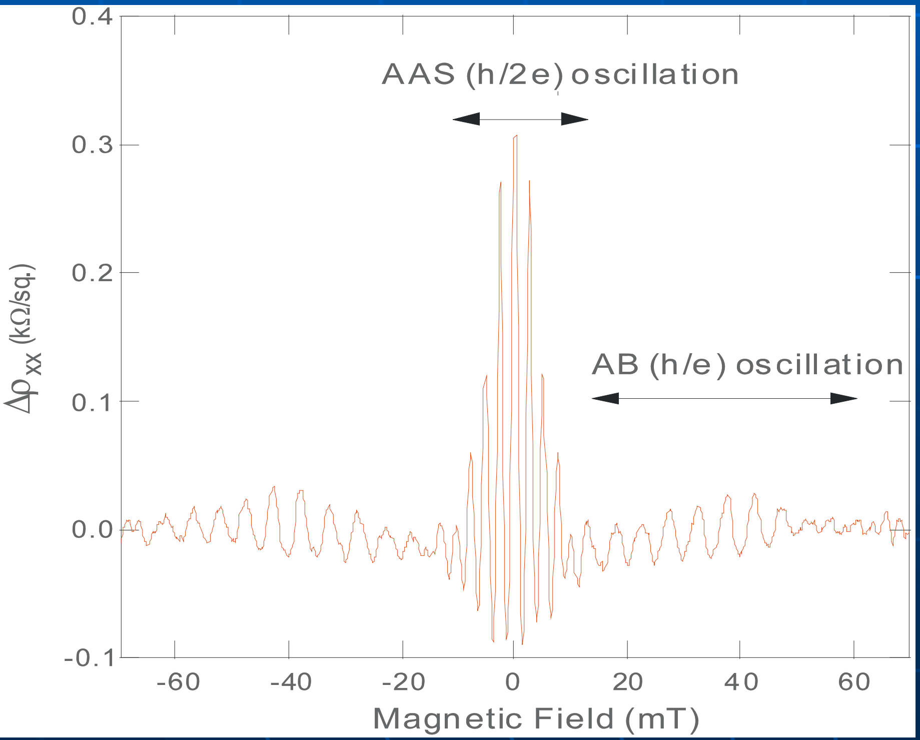




Mesoscopic Ring

$$|\Psi|^2 = |\Psi_+ + \Psi_-|^2$$
$$= |\Psi_+|^2 + |\Psi_-|^2 + \Psi_+\Psi_-^* + \Psi_+^*\Psi_-$$

quantum interference term



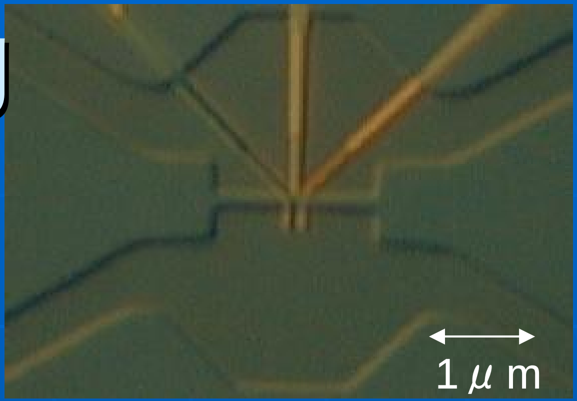
Interference between two electron waves that travel along the paths on both sides of a ring.

Electrical resistance changes periodically with the magnetic flux Φ piercing the ring.

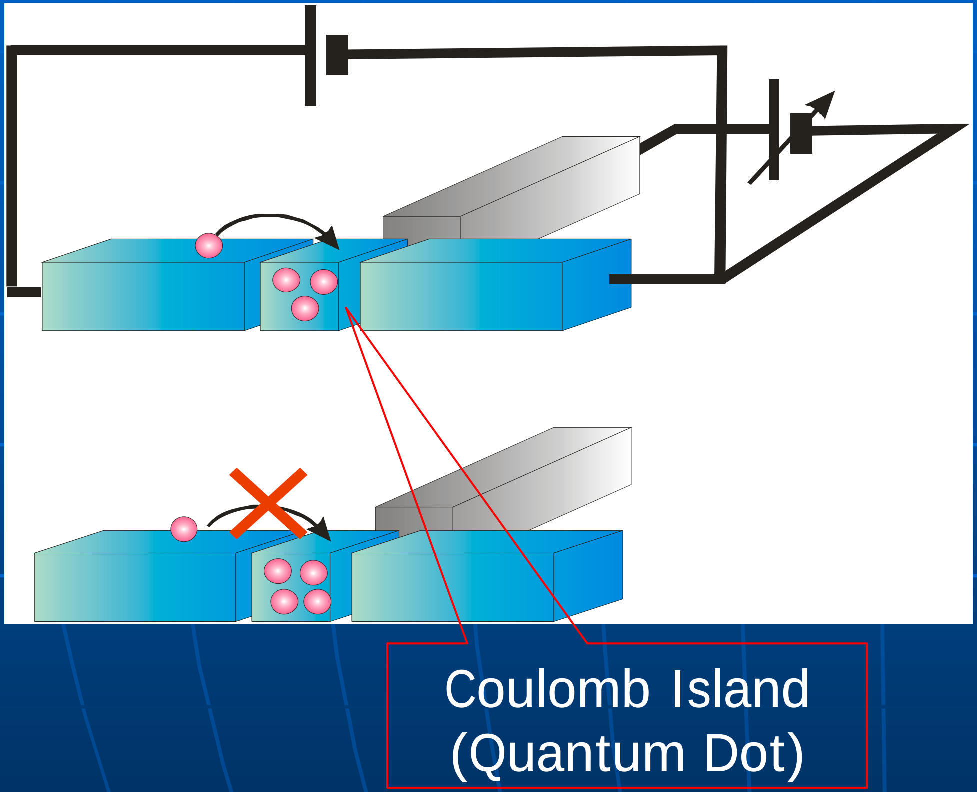
$$\phi_0 = \frac{h}{e} = 4.14 \times 10^{-15}\,\mathrm{Wb}$$

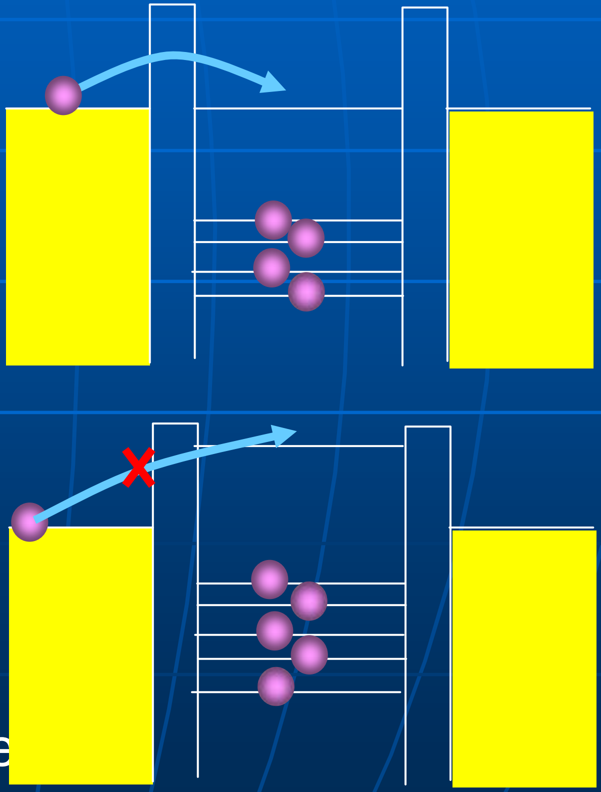
Flux Quantum

Single Electron Tunnelling



Nano-Tunnel Junction

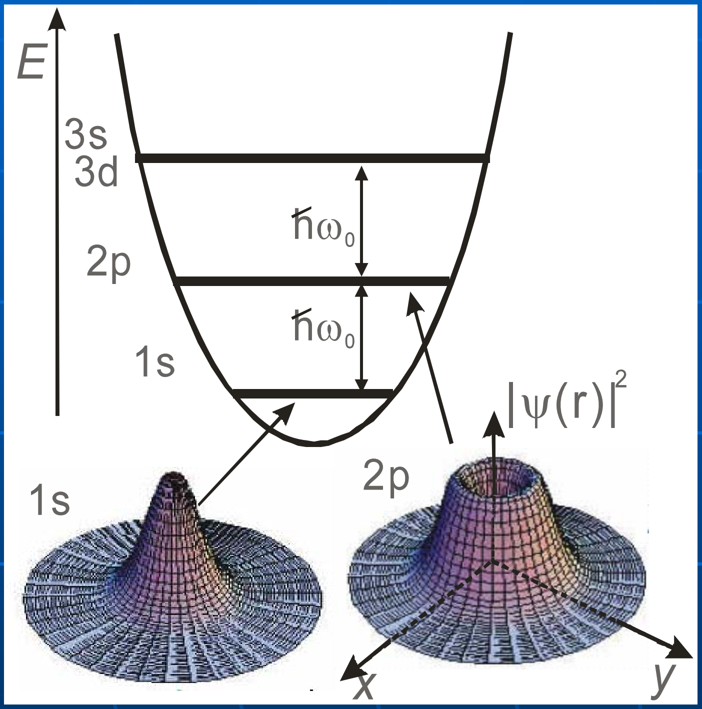


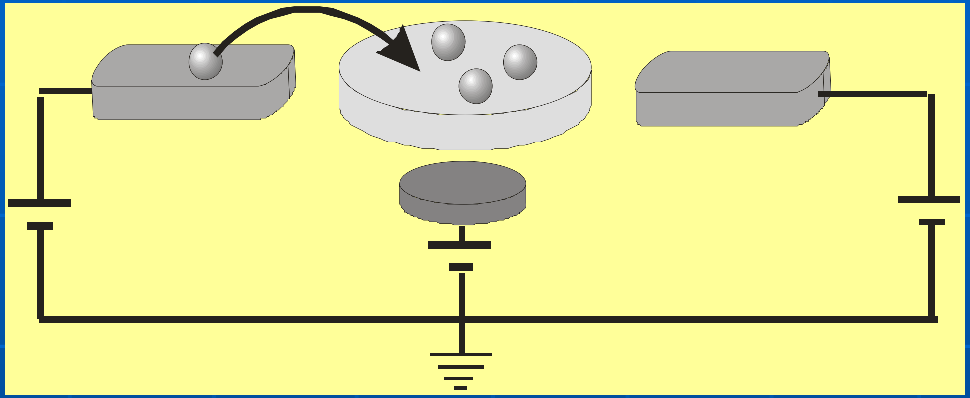


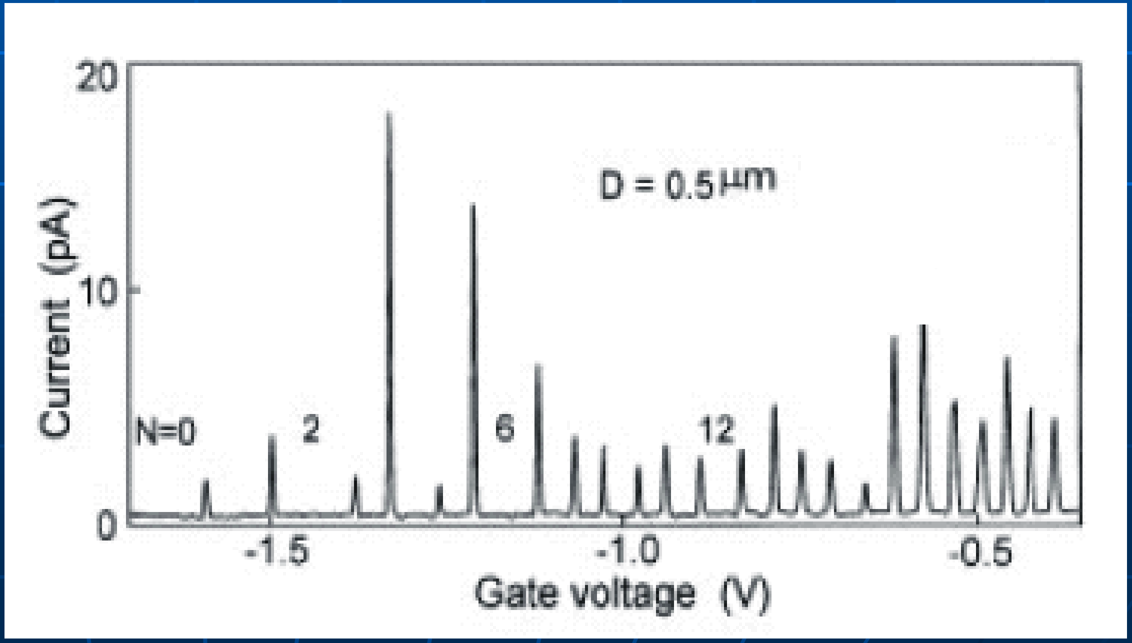
The electrostatic potential of the island increases as one electron tunnels into the island
=> next electron cannot enter the island

Coulomb Blockade

Quantum Dot (Artificial Atom)





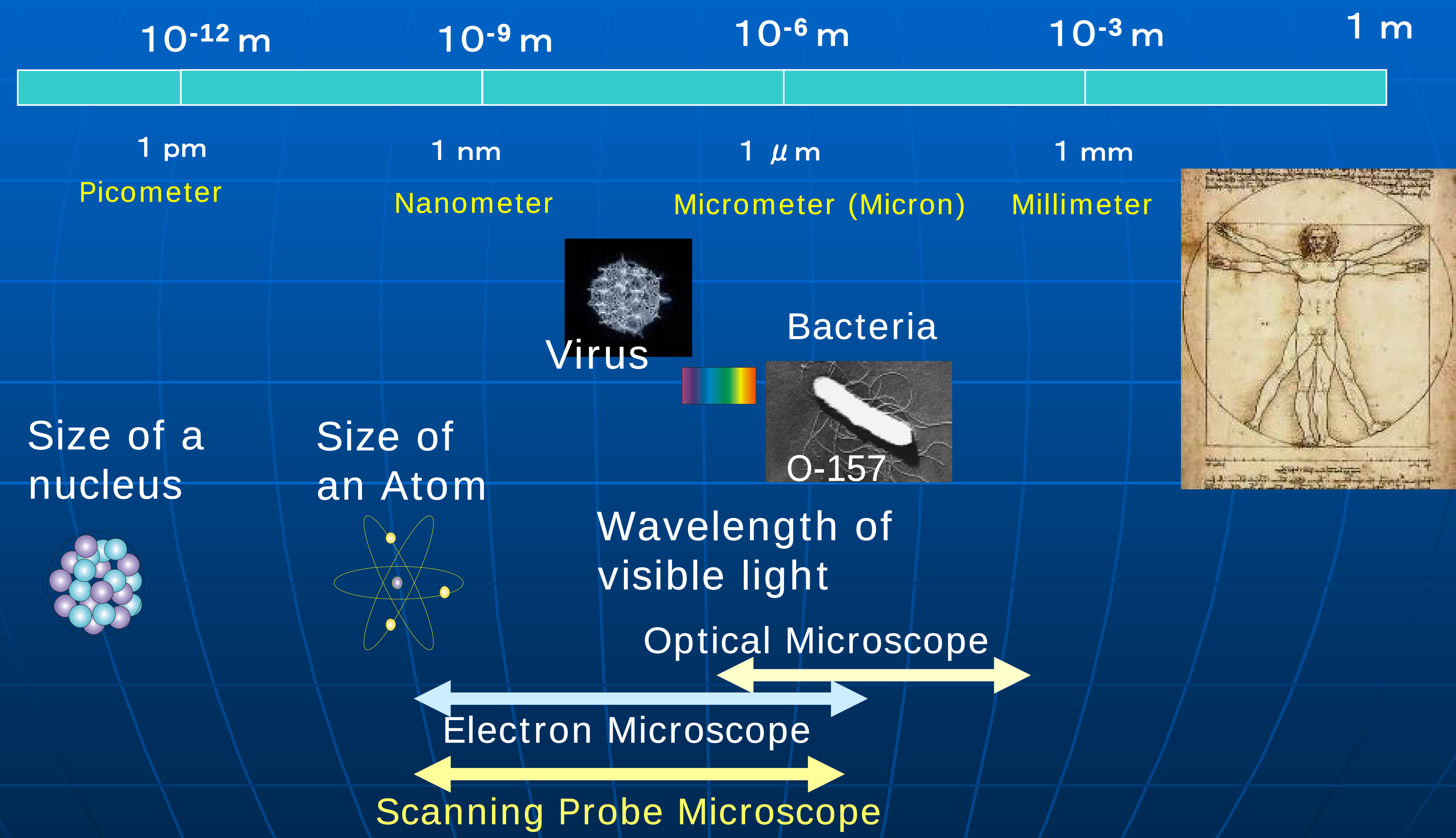


"Periodic table" of artificial atoms

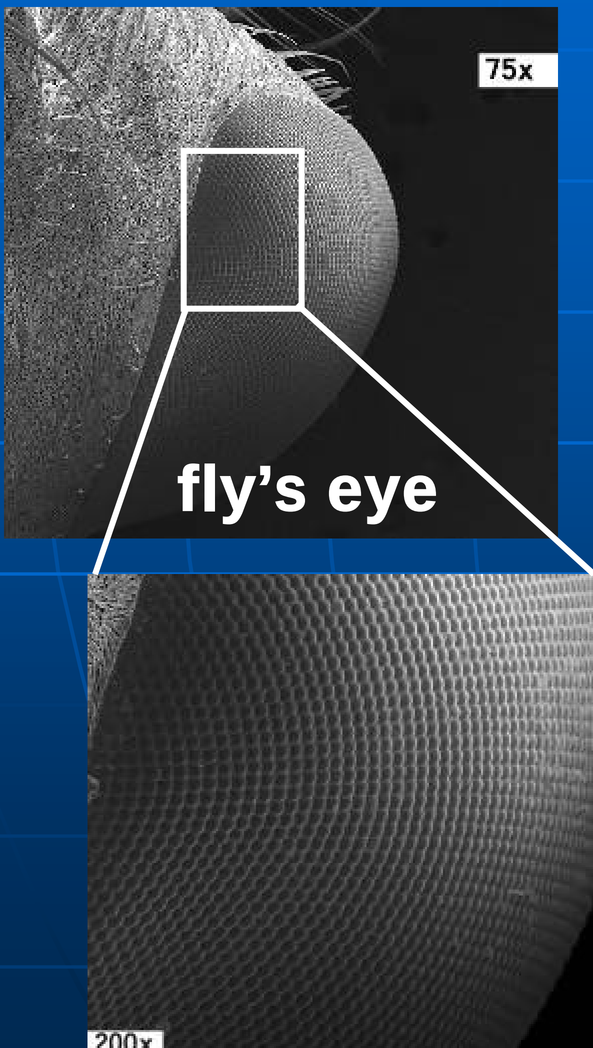
‡ Prof. Tarucha, Faculty of Science

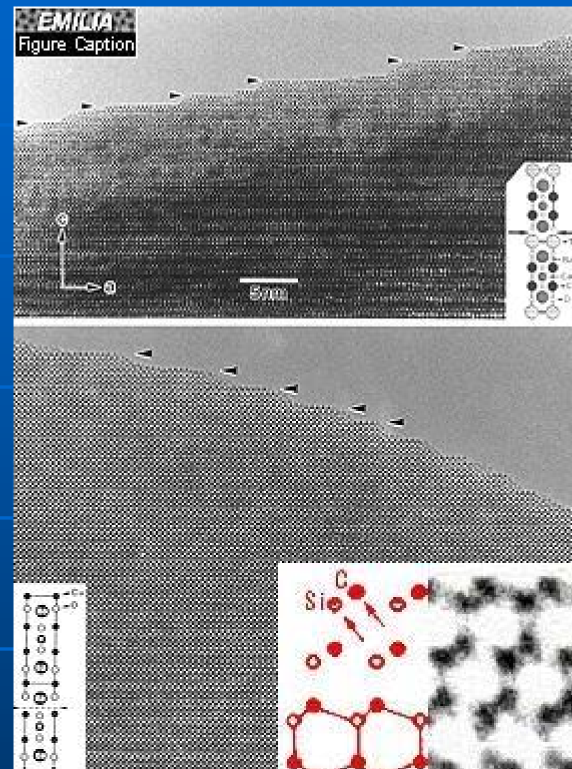
Observing Atoms
Manipulating Atoms

The Tale of Small World

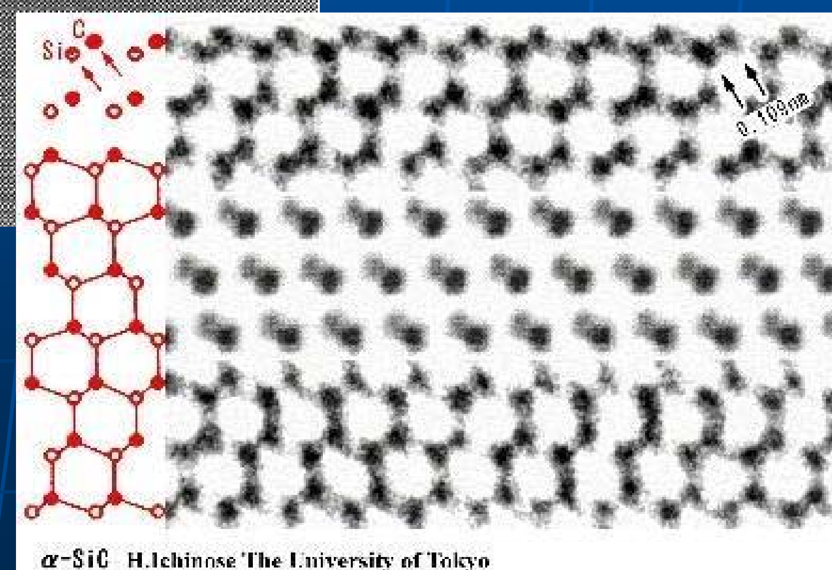


Seeing Small Objects : Electron Microscope





Observe the regular array of atoms with a high-resolution electron microscope



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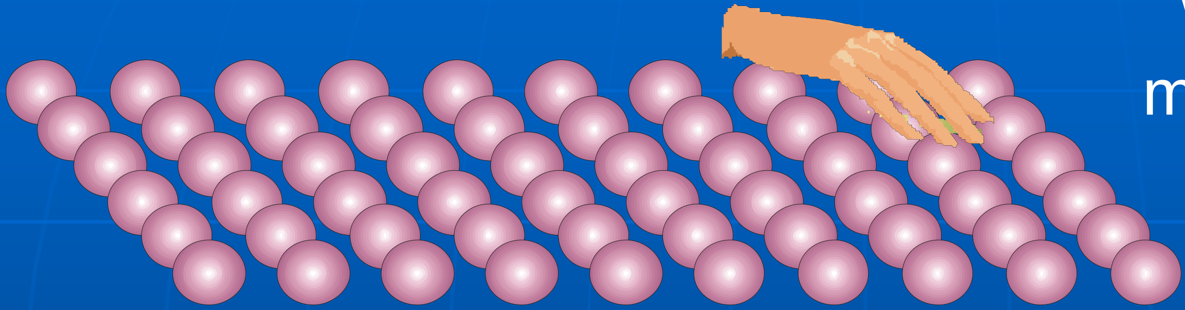
Observing the Arrangement of Atoms on a Solid Surface

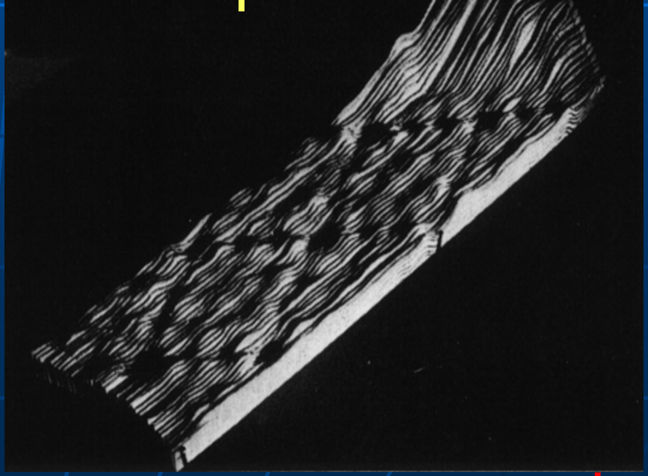


Were it an array of macroscopic objects you can see (or feel) the array by touching it

Would it be possible to achieve the same thing to an array of atoms?

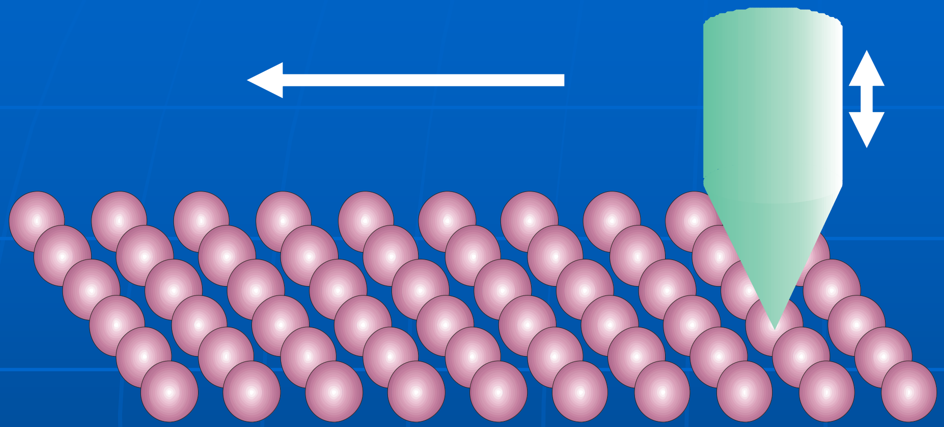
=> common sense tells us that it is "impossible"

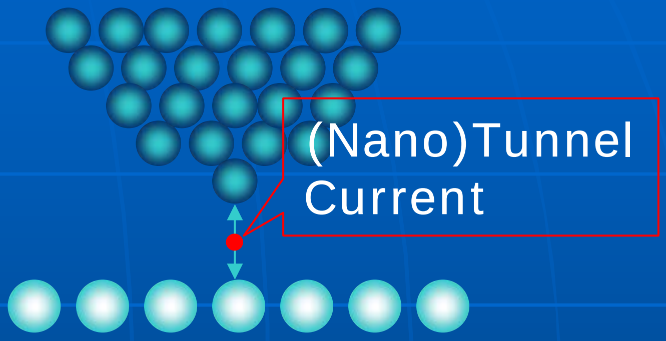
Scanning Tunnel Microscope
1984 Binnig and Rohrer



The first image of atomic arrangement on the surface of a silicon crystal.

Scanning Tunnel Microscope (STM)

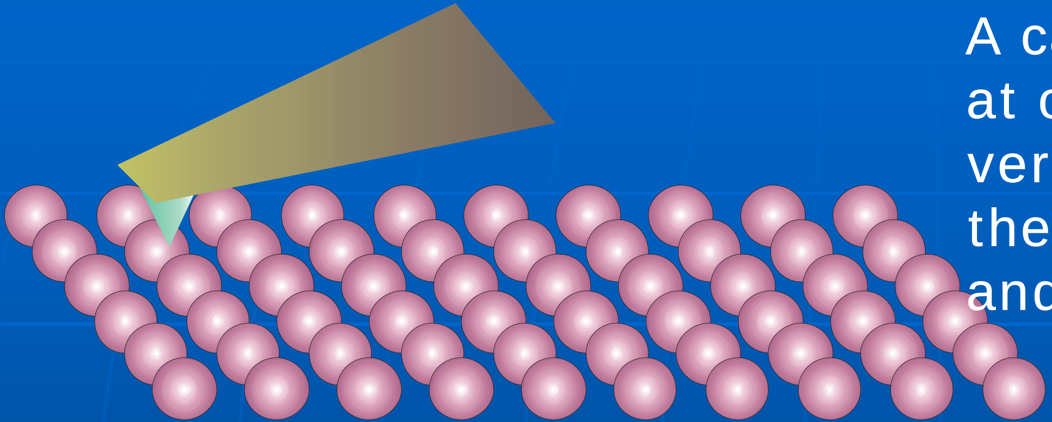




When atoms at the tip of the needle and atoms on the surface are brought to within ~1 nanometer, tunnelling current flows. By moving the tip laterally while keeping the tunneling current constant, one can detect the microscopic undulation due to the array of surface atoms.

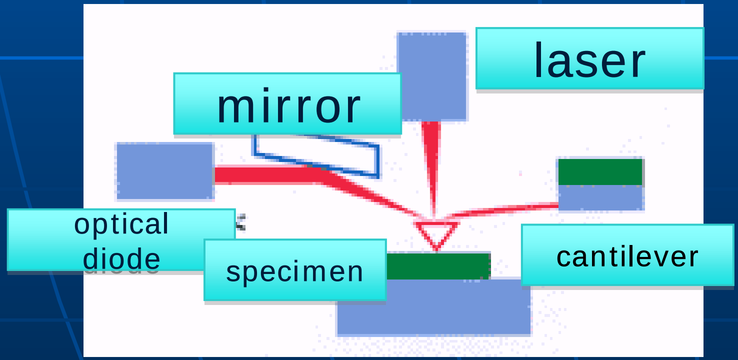
Problem of Stability
Mechanical Vibration
Electrical Noise

Atomic Force Microscope (AFM)

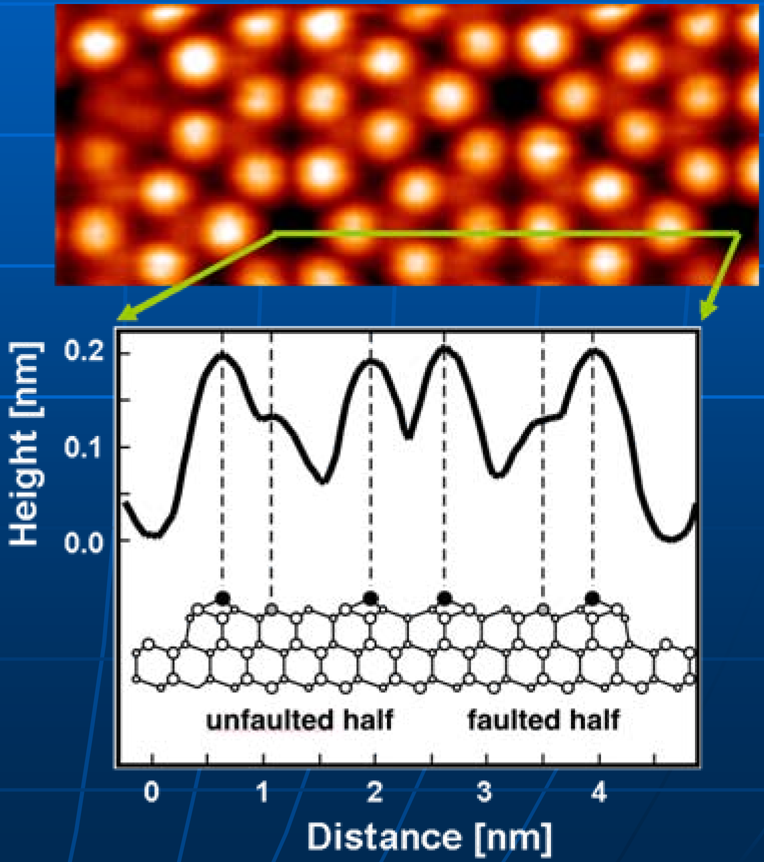


A cantilever (a flexible beam fixed at one end) probe is use to detect very weak force acting between the atom at the tip of the probe and atoms on the surface.

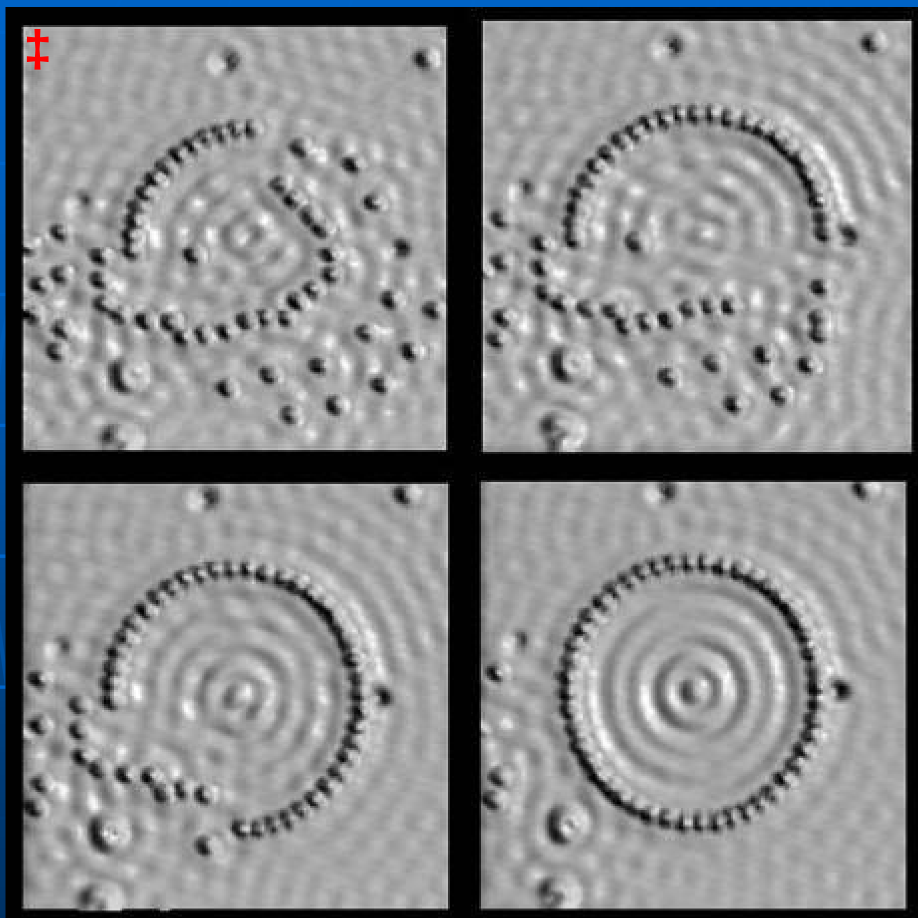
Detect the change in the atomic force as the tip is brought close to and scanned over the surface.



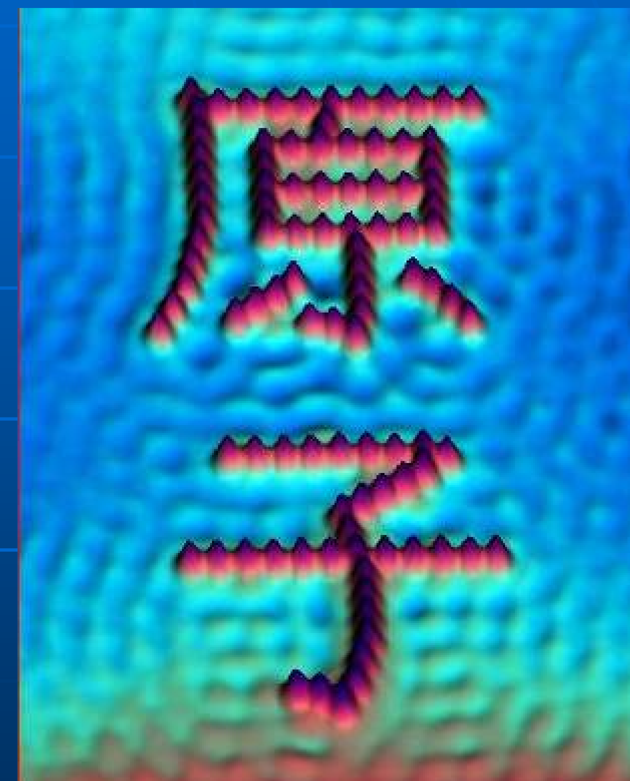
Deflection of the cantilever is detected by the change in the reflected laser beam.



Manipulating Atoms



IBM Almaden Lab
Eigler Group



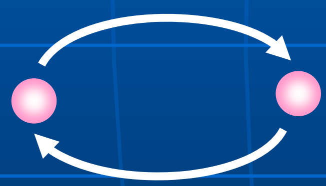
Artificial arrangement iron atoms on a copper surface.
The ripple-like feature reflects the wave interference of the surface electrons

Macroscopic Quantum Phenomena ---- Bose Condensation and Superconductivity

Quantum Mechanical Particles

Quantum mechanical particles of the same type are intrinsically indistinguishable.

Even when two particles are interchanged, the state remains the same as before.
The wave function, however, is multiplied by a numerical factor.



$$\Psi(\mathrm{b,a}) = C\,\Psi(\mathrm{a,b})$$

$$\Psi(\mathrm{a,b}) = C\,\Psi(\mathrm{b,a}) = C^2\,\Psi(\mathrm{a,b})$$

$$\Rightarrow \quad C^2 = 1$$

$$\Rightarrow \quad C = 1 \quad \text{or} \quad -1$$

Boson ($C = 1$)

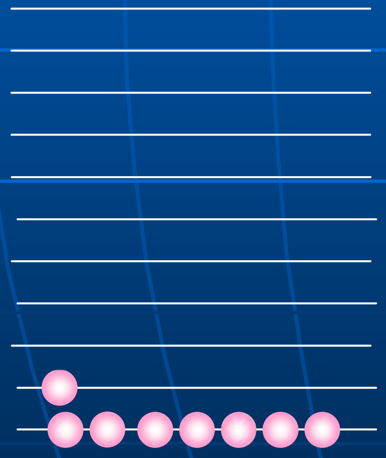
Fermion ($C = -1$)

Quantum Statistics

Bose Particle (Boson)

Spin: 0,1,...

$$\Psi(b,a) = \Psi(a,b)$$

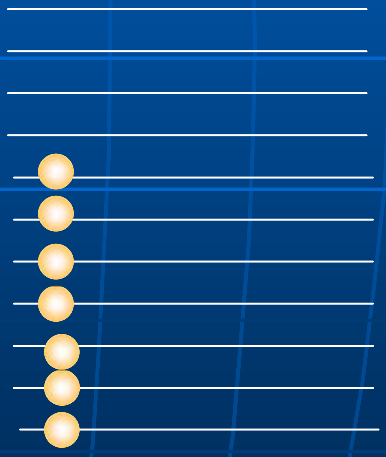


Any number of particles can occupy the same state

Fermi Particle (Fermion)

Spin: ½, 3/2, ...

$$\Psi(b,a) = -\Psi(a,b)$$



If a=b then

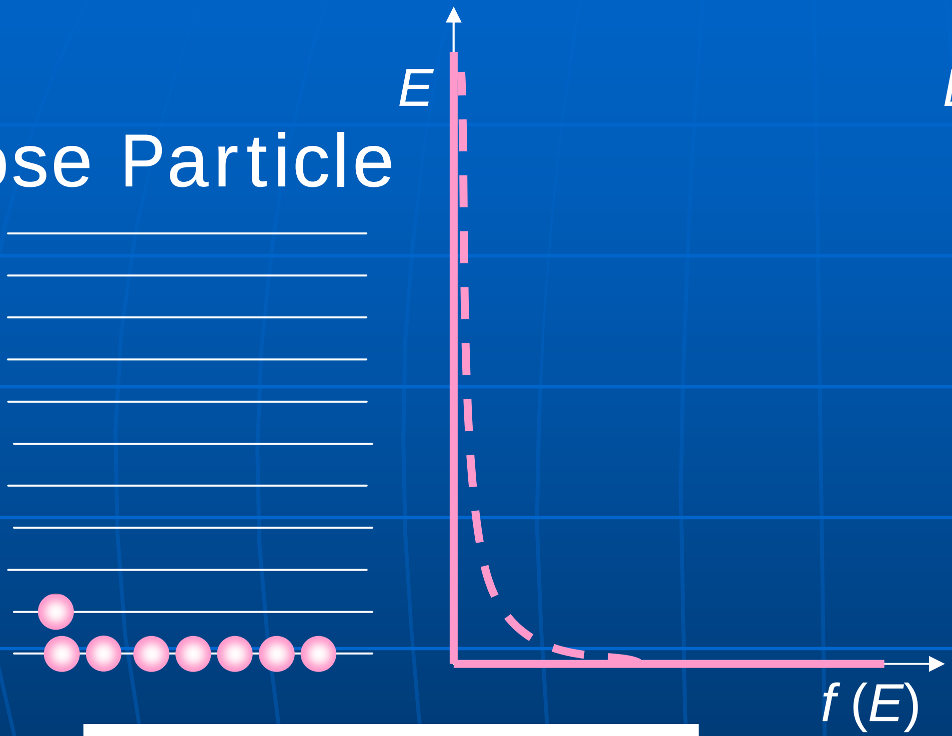
$$\Psi(a,a) = -\Psi(a,a)$$

$$\Rightarrow \Psi(a,a) = 0$$

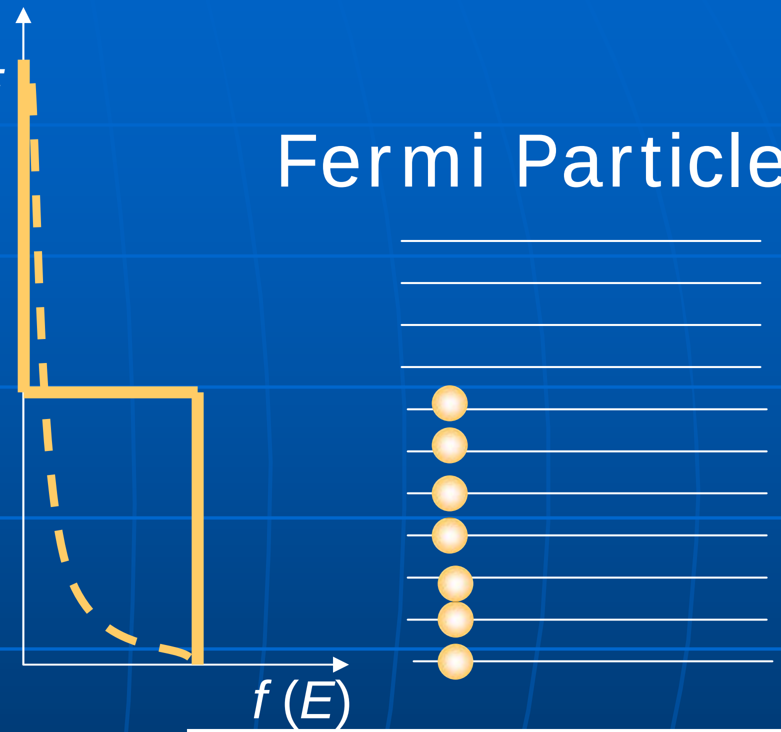
Only one particle is allowed in each state (Pauli Exclusion Principle)

Bose-Einstein Distribution and Fermi-Dirac Distribution

Bose Particle



Fermi Particle



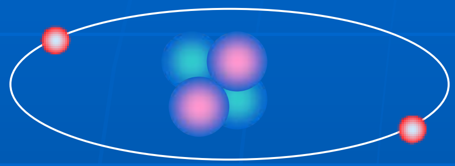
$$f_{\mathrm{BE}}(E) = \frac{1}{e^{(E-\mu)/k_{\mathrm{B}}T} - 1}$$

$$f_{\mathrm{FD}}(E) = \frac{1}{e^{(E-\mu)/k_{\mathrm{B}}T} + 1}$$

Either distribution is reduced to the classical Maxwell-Boltzmann distribution at high enough temperatures

$$f(E) = e^{-(E-\mu)/k_{\mathrm{B}}T}$$

Helium Isotopes

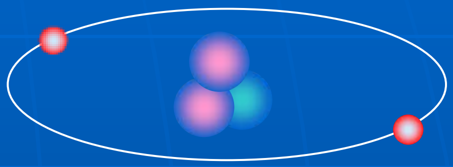


^{4}He

2 Protons
2 Neutrons
2 Electrons

Total Spin = 0

Bose Particle



^{3}He

2 Protons
1 Neutrons
2 Electrons

Total Spin = 1/2

Fermi Particle

Making Ultra-Low Temperatures

Ultra-low temperatures are required to observe phenomena associated with quantum statistics

Cut-away view of a liquid helium vessel

Liquid Nitrogen 77K

Liquid Helium (^{4}He) 4.2K

forced evaporation by pumping

~1.2K

Liquid Helium (^{3}He) 3.2K

forced evaporation by pumping

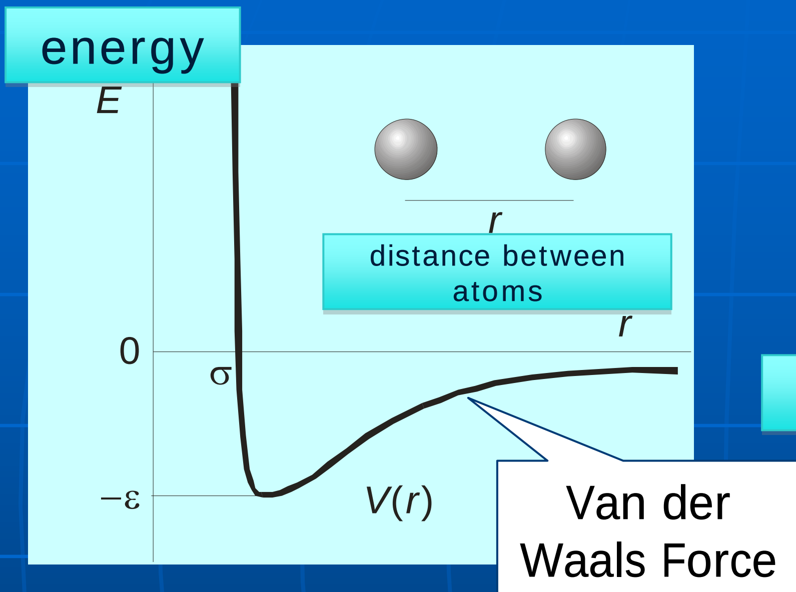
~0.3K

^{3}He-^{4}He Dilution Refrigerator ~ mK

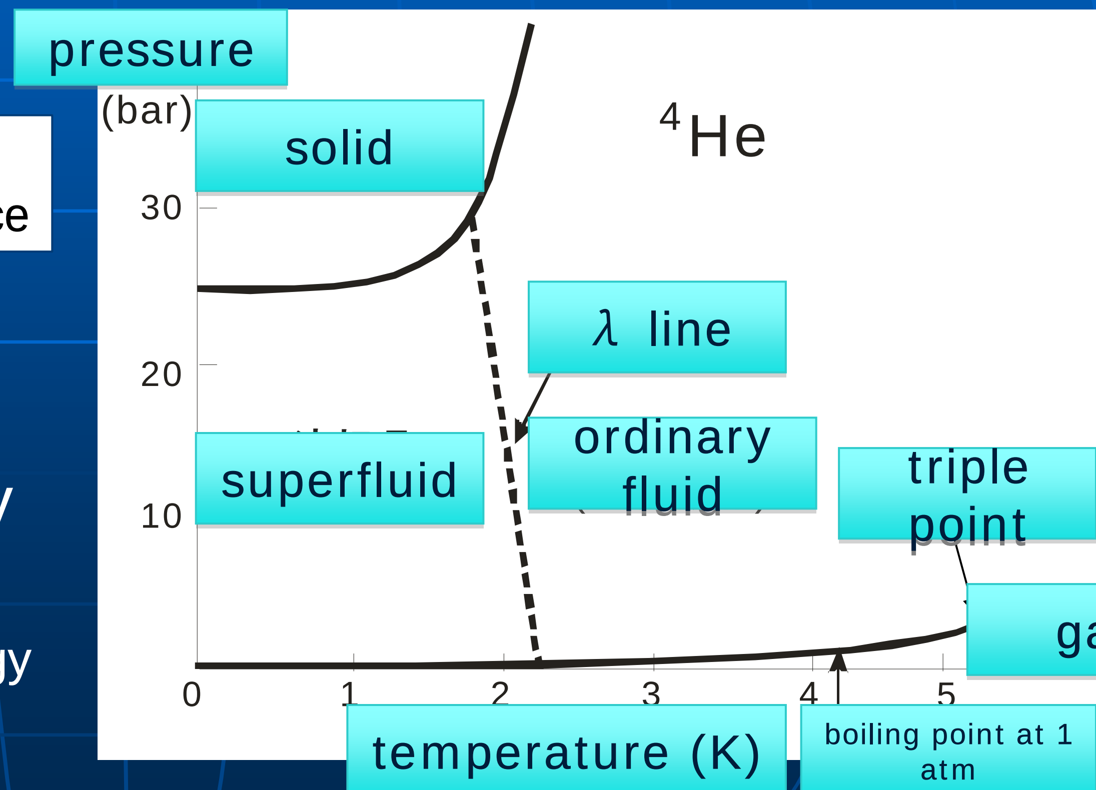
Adiabatic Nuclear Demagnetization

~ μK

Phase Diagram of Helium



Helium (at ambient pressure) does not solidify even at absolute zero
=> Quantum Liquid



Helium atoms are light.

Helium atoms interact only weakly with each other.

Kinetic energy > Interaction energy

Superfluidity of Liquid Helium

The University of Tokyo Cryogenic Research Center ‡

Superfluidity of Liquid Helium

figure 1

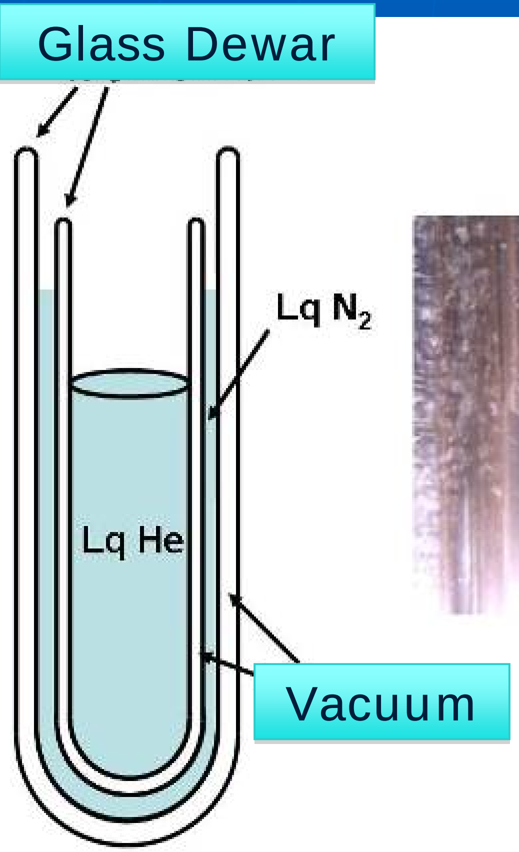
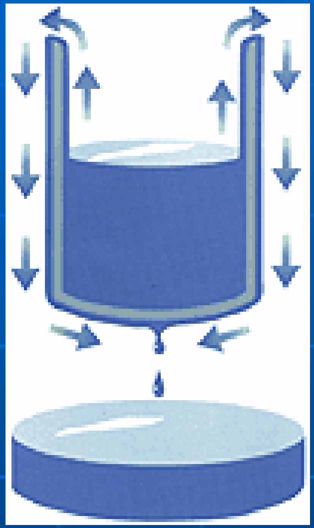
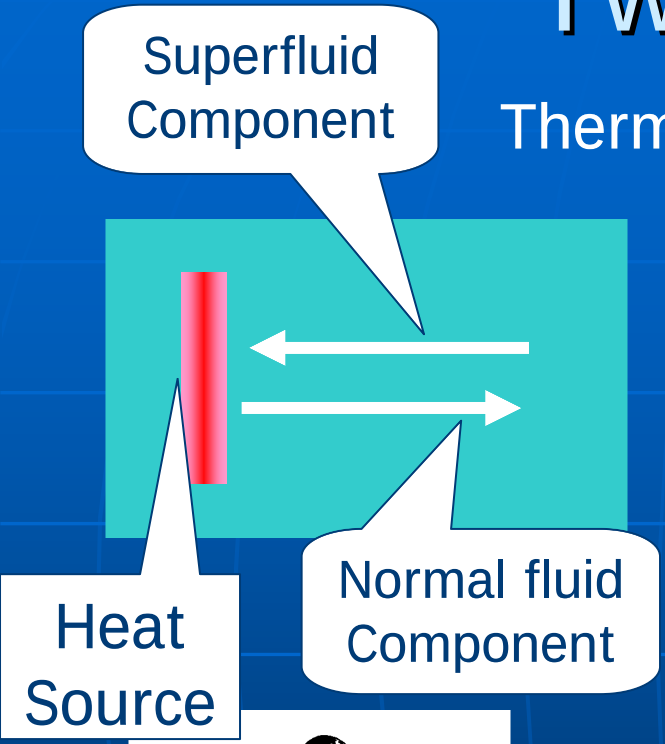


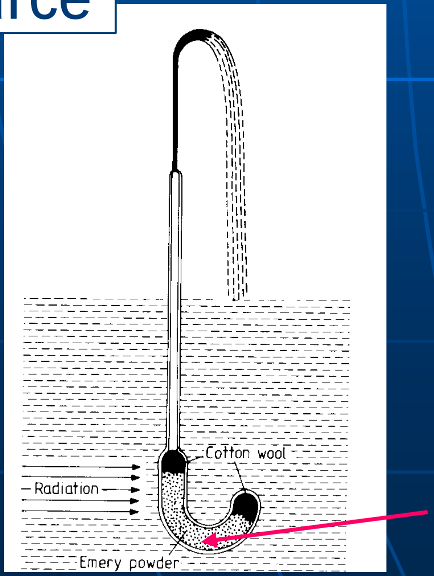
photo 1



Two-Fluid Model



Thermomechanical Effect (Internal Convection)



Fountain Effect

the University of Tokyo Cryogenic Research Center

Superfluid component flows without friction through a capillary (superleak) which is too narrow for the normal fluid to pass.

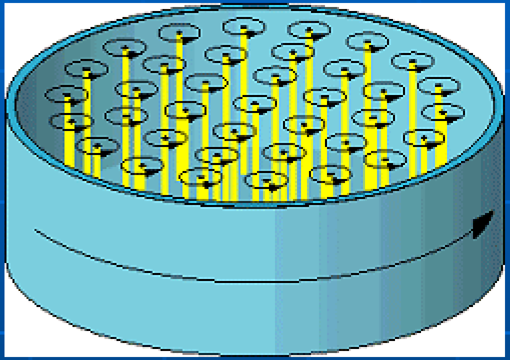
Quantum Vortex

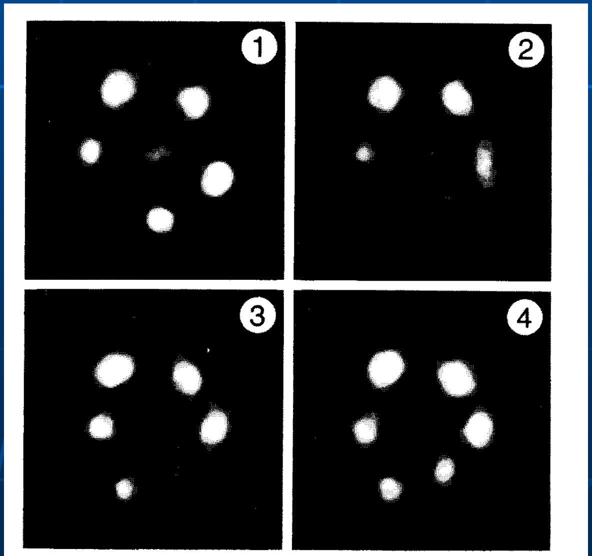
Macroscopic Wave Function $\Psi = \Psi_0 e^{i\theta}$

Quantization of Circulation

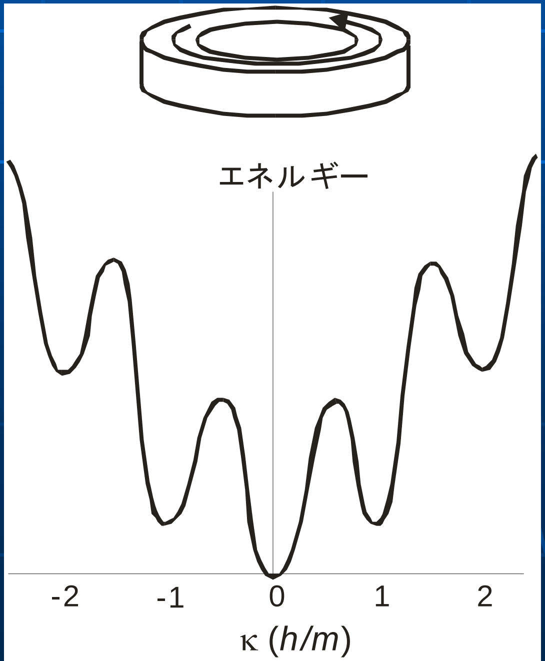
$$\kappa \equiv \oint_C \mathbf{v}_s \cdot d\mathbf{s} = \frac{\hbar}{m} \oint_C \nabla \theta \cdot d\mathbf{s} = \frac{\hbar}{m} 2\pi n = n \frac{h}{m}$$

Rotating Bucket Experiment

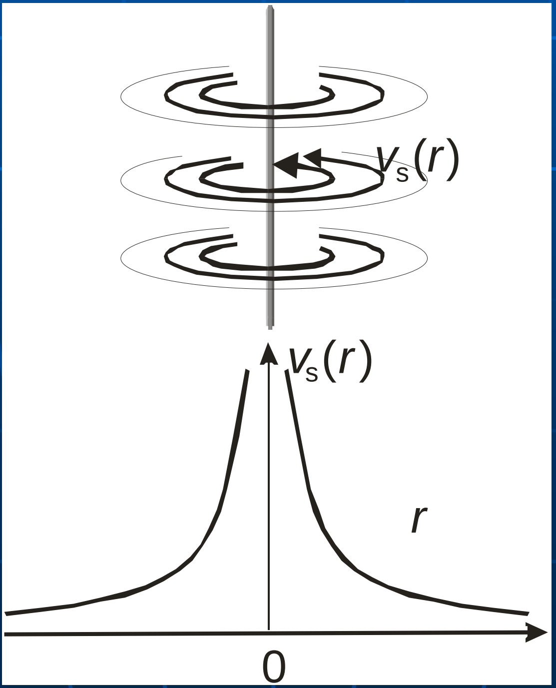




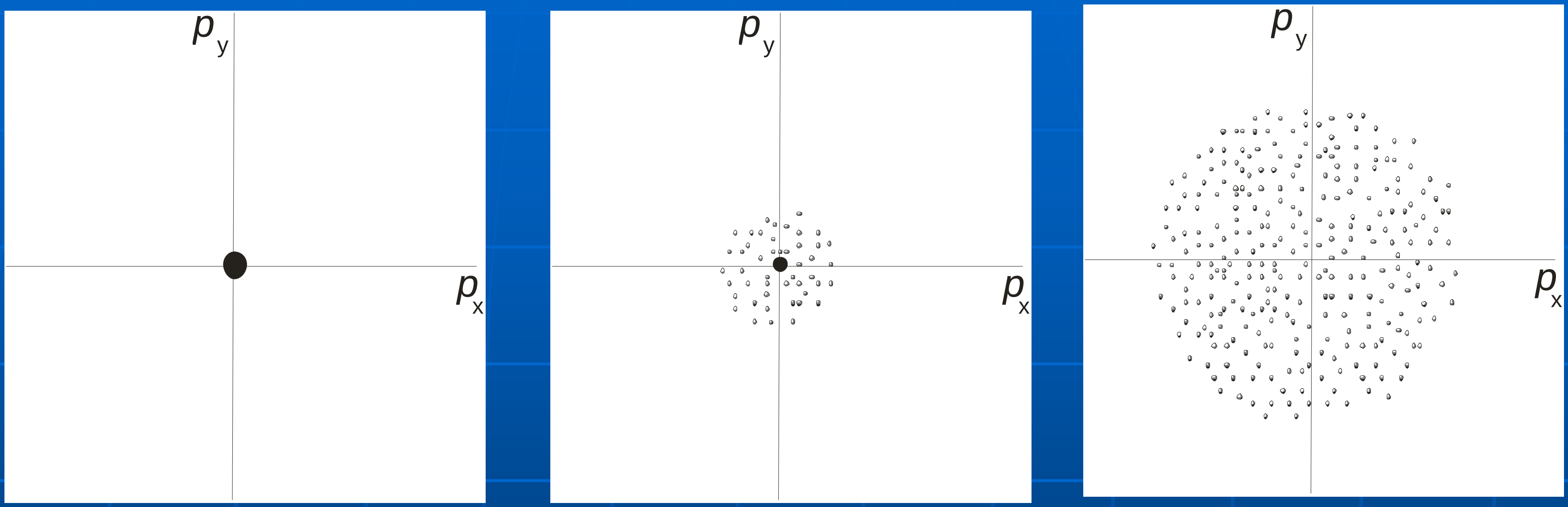
Permanent Current



Quantum Vortex



Bose-Einstein Condensation



$T = 0$ $\quad$ $T = T_{BE}$ $\quad$ $T > T_{BE}$

Thermal de Broglie Wavelength

$$\lambda_T = \left(\frac{2\pi \hbar^2}{m k_B T} \right)^{1/2}$$

Bose condensation occurs when the thermal de Broglie wavelength becomes comparable to the interparticle distance

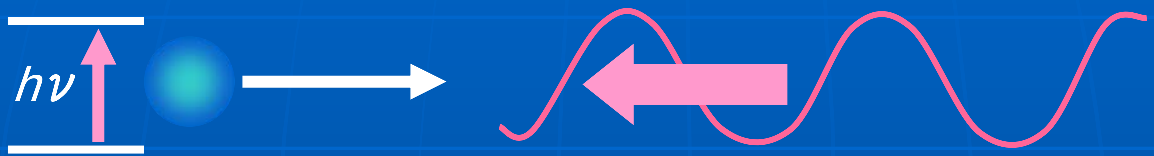
$$\lambda_T \approx n^{-\frac{1}{3}}$$

$$T_{BE} = \frac{2\pi\hbar^2}{m k_B} \left(\frac{n}{2.612} \right)^{\frac{2}{3}}$$

Laser Cooling of Atomic Gases

Collect and cool an atomic (*e.g.* Rb) gas (vapor) in a trap

Doppler Cooling



・Use a laser light with frequency slightly below the resonance frequency of the atoms
・For an atom travelling in the opposite direction to the light, the Doppler shifted light frequency becomes closer to the resonance frequency, so the absorption probability is higher. The momentum of the atom is reduced by absorbing the incident light (photon).
・When the atom reemits the light, it does so isotropically, so that on average the atom is slowed down.
・By a set of six laser beams along the x-, y-, z- axes, Doppler cooling is achieved for all directions.
・With this technique, temperature on the order of ~100μK is attained.
=> Further reduction of temperature by 3 to 4 orders of magnitude is needed.

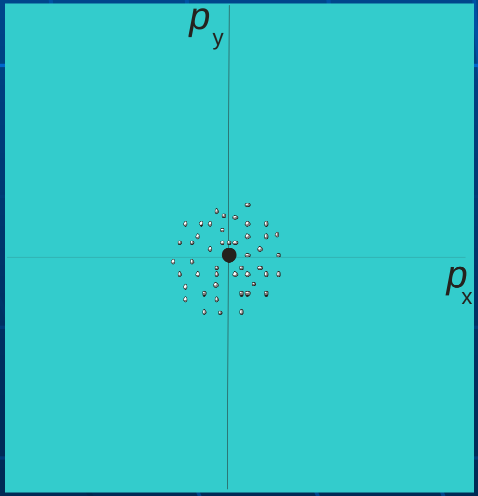
Bose-Einstein Condensation of Atomic Gases

Reducing the temperature and
achieving the conditions of Bose-Einstein Condensation by
Evaporative Cooling of atomic gas
cooled in a magnetooptical trap
$T \sim 10^{-7}$K

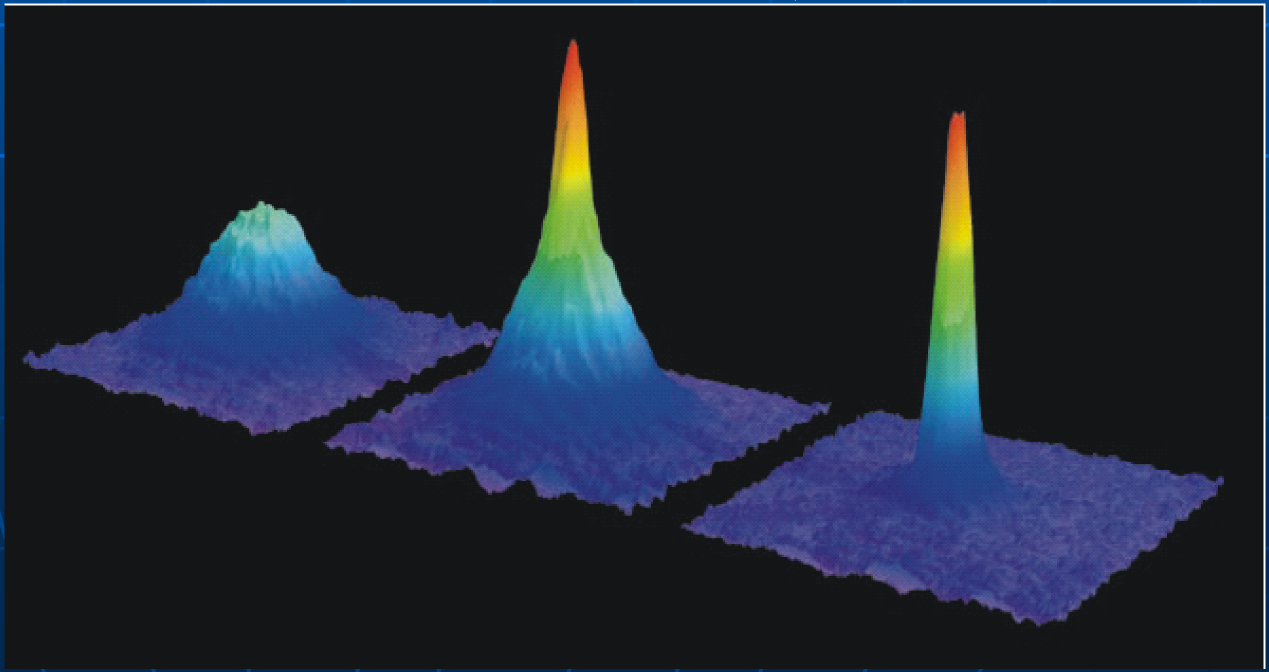
$$\lambda_T \approx n^{-1/3}$$

Upon turning off the trap potential, the cloud of cold atoms drops with gravity, and at the same time swells according to the velocity distribution.

‡ http://cua.mit.edu/ketterle_group/research.htm

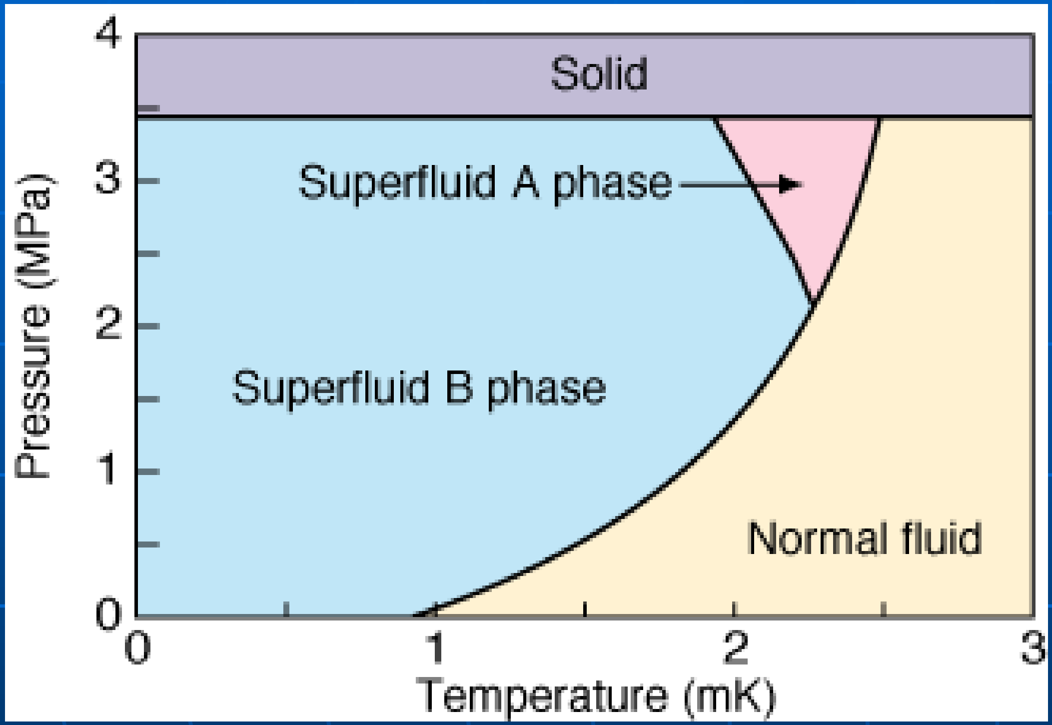


$T = T_{BE}$



Superfluidity of Liquid ^3He

Phase diagram of ^3He

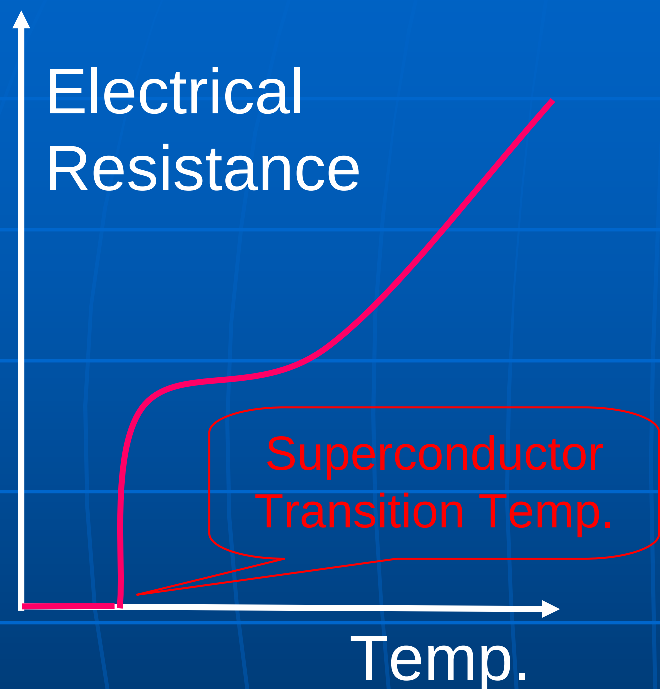


Being Fermi particles, ^3He atoms do not undergo Bose condensation per se. However, they can become superfluid by forming pairs. (This is the same mechanism as superconductivity)

^3He becomes a superfluid at an ultra-low temperature of ~2mK

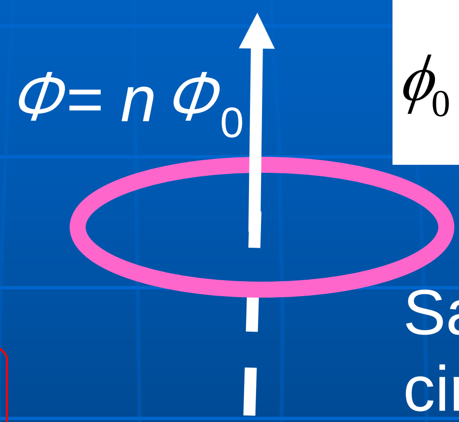
Basic Properties of Super-Conduction

Perfect Conductor (Zero Resistance)



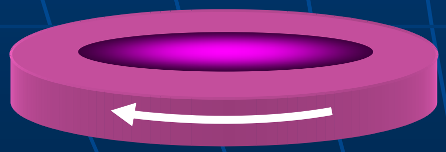
Quantization of Magnetic Flux

$$\phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\,\mathrm{Wb}$$

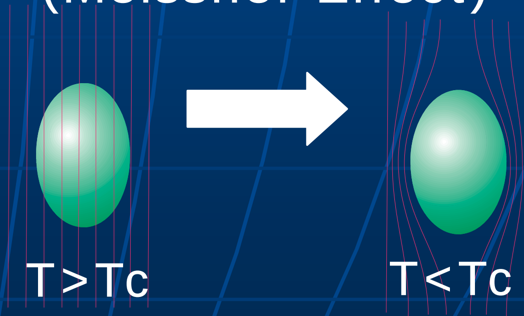


Same as quantization of circulation in superfluid

Persistent Electrical Current

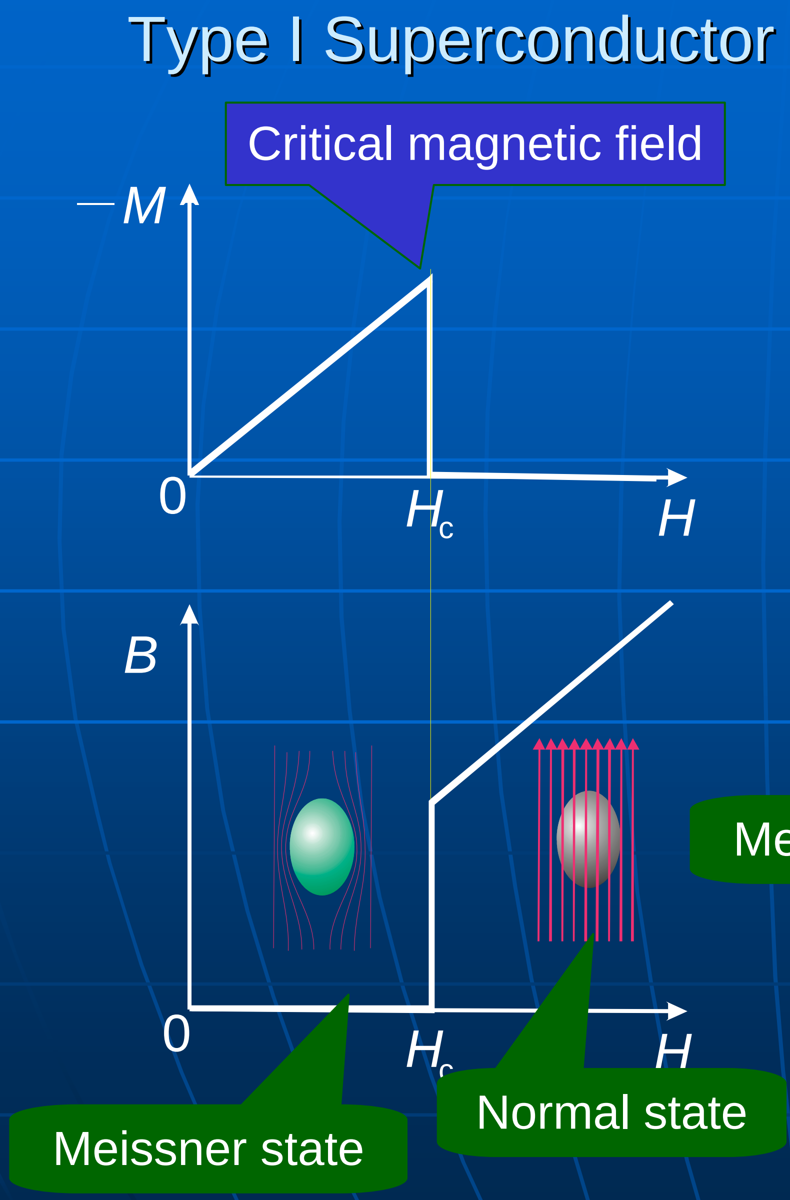


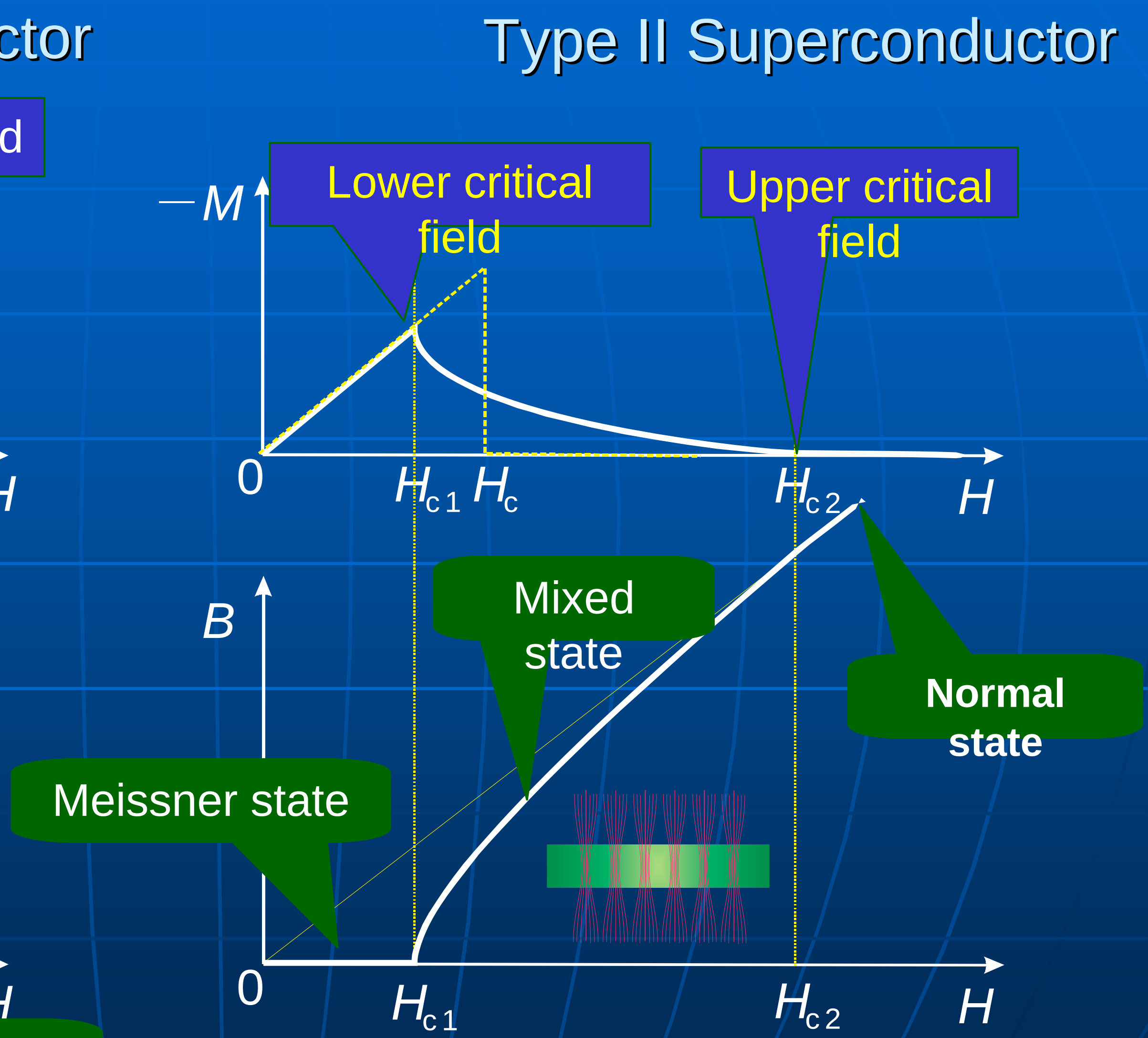
Perfect Diamagnetism
(Meissner Effect)



Magnetic flux is expelled from a superconductor

Type I and Type II Superconductor

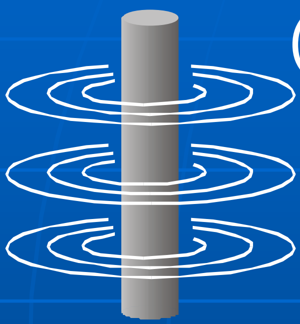




Superconducting materials of practical use are type II superconductors.

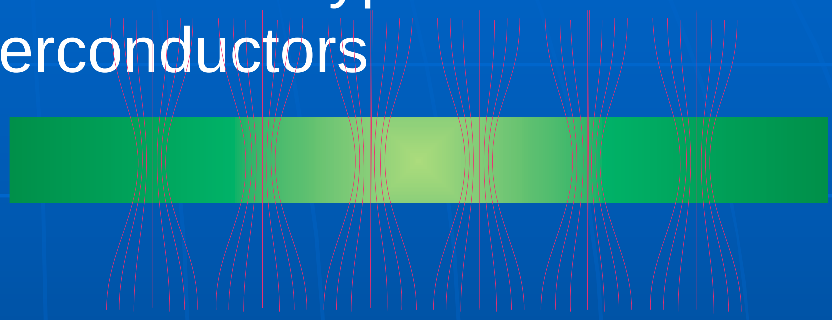
Quantum Magnetic Flux in Type II Superconductors (Vortex Line)

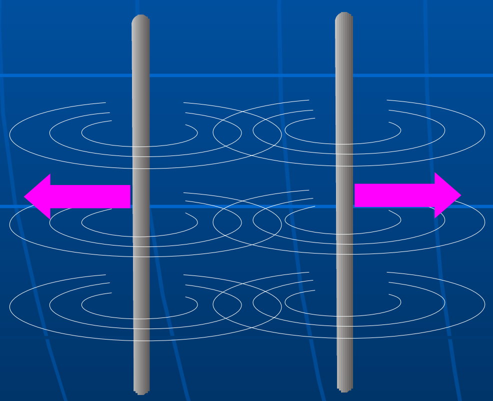
Quantum magnetic flux
(Vortex line)



$$\phi_0 = \frac{h}{2e} = 2.07\times10^{-15}\,\mathrm{Wb}$$

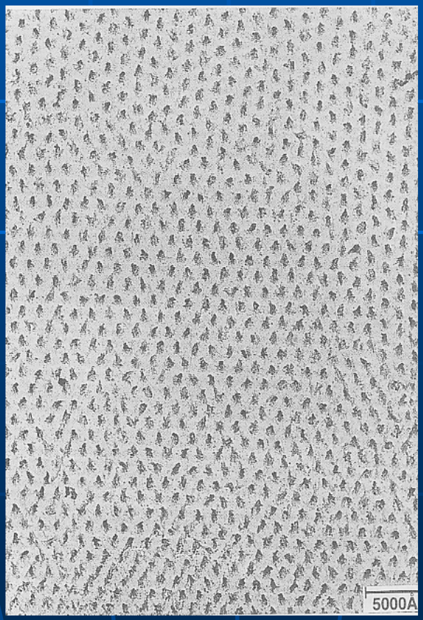
Mixed state of type II superconductors



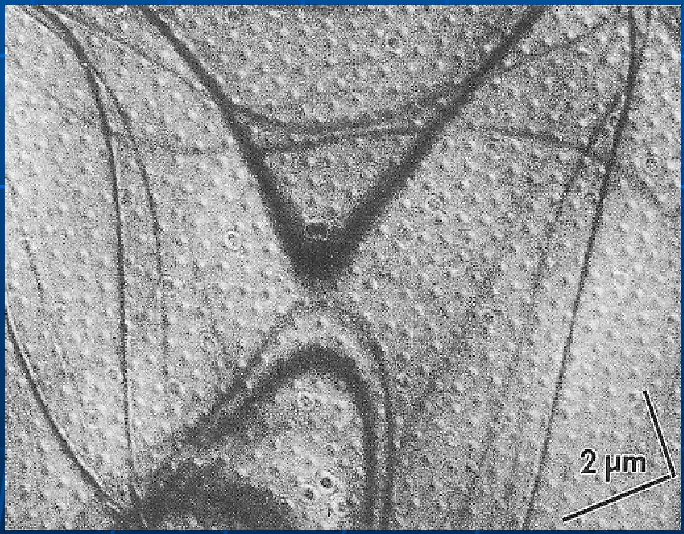


Repulsive force acts between vortex lines

Triangular lattice



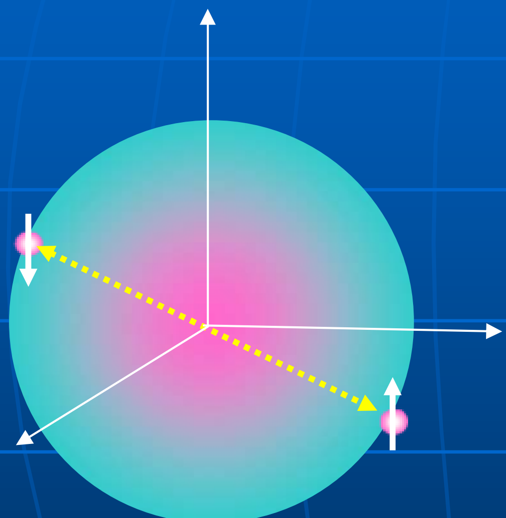
‡ Bitter Method
(Essmann & Traueble,1968)



‡ Lorentz Microscope
（Tonomura Akira, 1992)

Mechanisms of Superconductivity

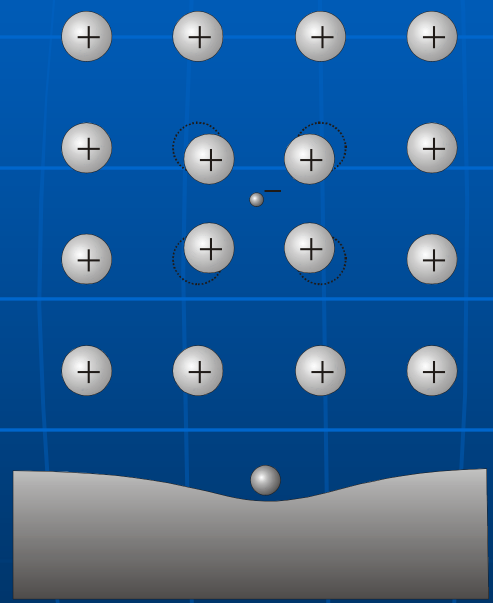
Cooper Pair Configuration

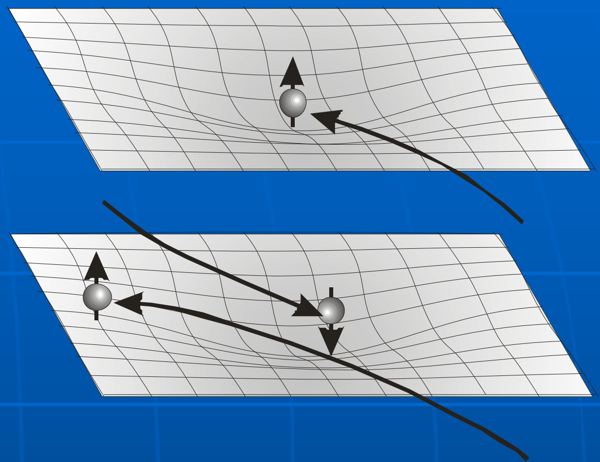


If attractive force acts between two electrons on the Fermi surface, they form a bound state (Cooper Pair).

Origin of Attraction?

Electron Particle Interaction



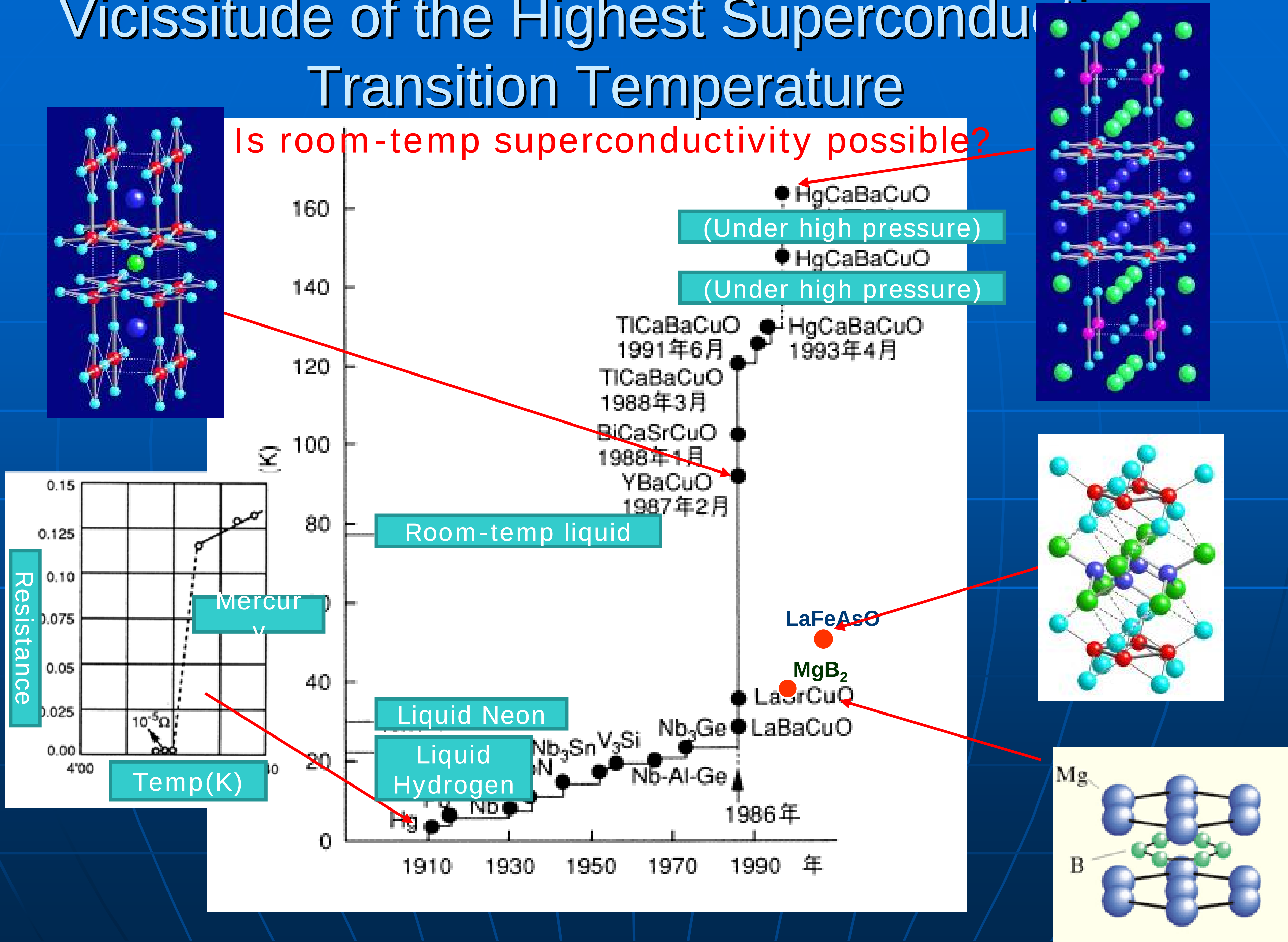


If the inter-electron attractive force mediated by electron-lattice interaction surpasses the repulsive Coulomb force, the net attractive inter-electron interaction results in Cooper pair formation.

Superconducting Transition Temperature

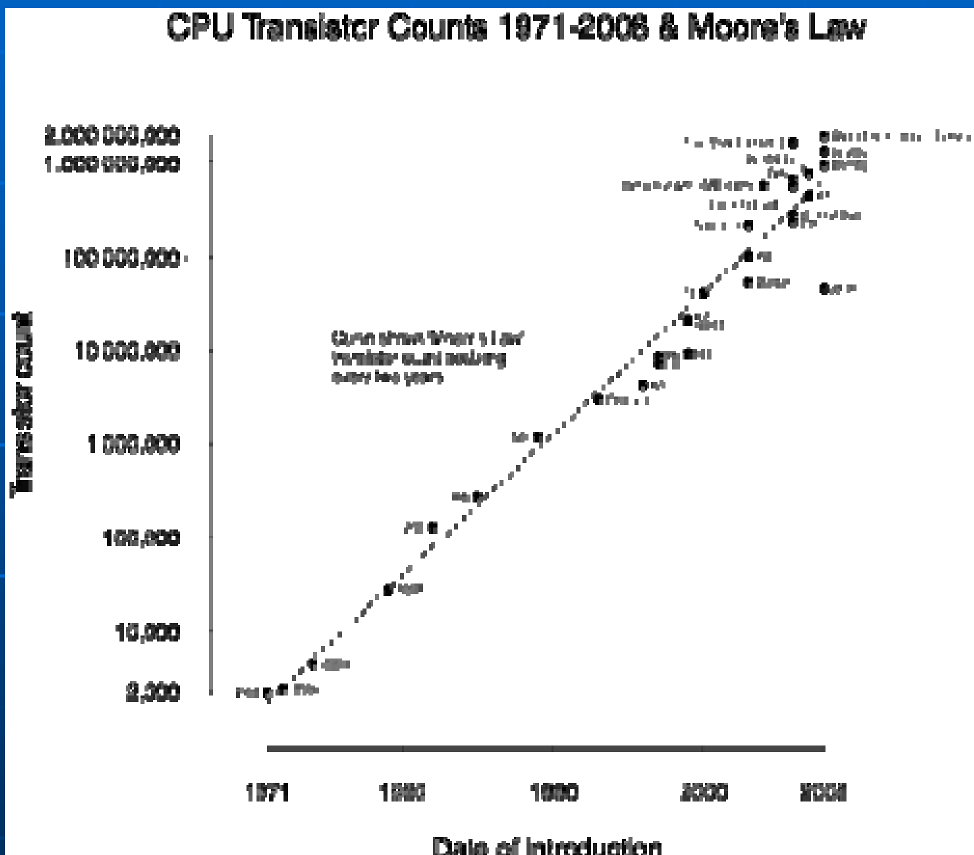
$$T_c = 1.14\Theta_D \exp\left(-\frac{1}{N(0)V}\right)$$

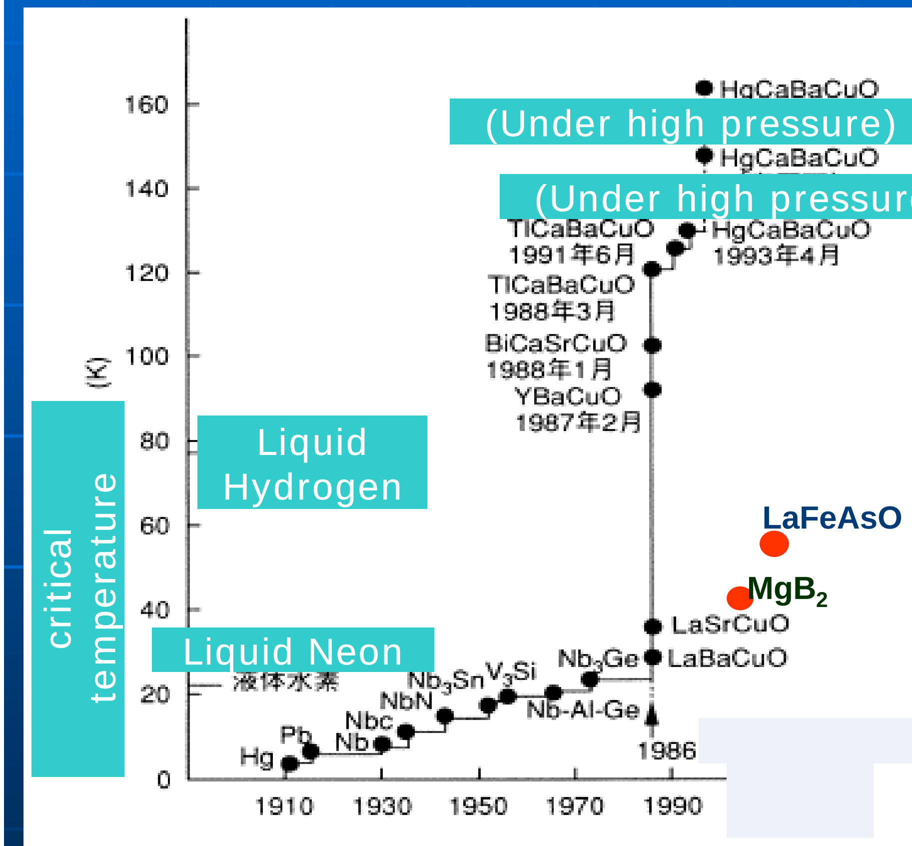
Vicissitude of the Highest Superconducting Transition Temperature



Prediction of the Future

Technological Advances vs. Scientific Developments





Roadmap ⇒
Self-fulfilling Prediction

It is impossible to foretell the development of science (discovery, in particular).

Summary

- Characteristic of Quantum Mechanics
  - Quantum interference, Tunnel effect
- Mesoscopic Physics
  - Quantum conductance, e^2/h-physics
  - Quantum Interference AB Effect
  - Single electron tunneling Effect
- Nanotechnology and nanoscience
  - Observing atoms, Manipulating atoms
  - Scanning probe microscope
- Macroscopic Quantum Phenomena
  - Superfluidity
  - Bose Condensation
  - Superconductivity

The Role of Materials Science and Condensed Matter Physics

- Understanding the diverse properties of various substances based on the basic principles of physics (quantum mechanics and statistical mechanics)
- Phase transition and emergent phenomena:

  seeking universality and unity in diversity and complexity
- Performing experiments that approach the essence of quantum mechanics
- Toward the construction of "materials perspective": Some of the concepts formed share commonality with other subfields of physics, such as elementary particle physics or astrophysics.
- Forms the basis of engineering

The Role of Materials Scienc… nd Condensed Matter P… sics

- Understanding the diverse pro… s of various substances based on the basic princi… physics (quantum mechanics and stati… mechanics)
- … transiti… d emergent phenomena:

  seeking … sality and unity in diversity and complexity
- Performing … ments that approach the essence of quantum mechani…
- Toward the construction … aterials perspective": Some of the concepts formed share co… ality with other subfields of physics, such as elementary par… hysics or astrophysics.
- Forms the basis of engineering

There are so many interesting things!

Bilateral Pursuit of Humanities and Sciences!